SAMPLE PAPER 1

SECTION A

Q1. Let $R = \{(1,1)(2,2)(1,2)\}$ be the relation on the set $A = \{1,2,3\}$ then R is

(a) Reflexive only (b) Symmetric only (c) Transitive only (d) None

Q2. If k is scalar and A is a square matrix of order $n \times n$, then $|kA|$.

(a) $k|A|^n$ (b) $k|A|$ (c) $k^n|A|^n$ (d) $k^n|A|$

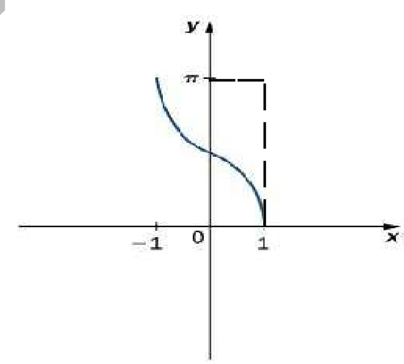
Q3. Value of $\sin^{-1}\sin\left(\frac{5\pi}{6}\right)$ is

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Q4. Identify the function shown in the graph :

(a) $\sin^{-1} x$ (b) $\cos^{-1} x$

(c) $\sec^{-1} x$ (d) $\tan^{-1} x$



Q5. The point of discontinuity, if any, for the function $f(x) = |x-5|$ is :

(a) $-\frac{1}{2}$ (b) 5 (c) -5 (d) No point of discountinuity

Q6. If $f(x) = \log x$, then $f(x)$ is :

(a) always increasing on domain
(b) always decreasing on domain
(c) both increasing and decreasing on domain
(d) neither increasing and decreasing on domain.

Q7. $\frac{d}{dx}\left[\sin^{-1}(\sin 2x)\right]$ is equal to :

(a) -2 (b) 1 (c) 2 (d) None

Q8. The point which does not lie in the half plane $2x + 3y - 15 < 0$ is

(a) $(0,2)$ (b) $(2,1)$ (c) $(3,5)$ (d) $(3,1)$

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Q9. Let A and B two events such that $P(A) = 0.6, P(B) = 0.2$, and $P(A/B) = 0.5$, then $P(A \cup B)$ is equal to

(a) 1/10 (b) 3/10 (c) 7/10 (d) None

Q10. Value of $\tan\left(\cos^{-1}\frac{4}{7}\right)$

(a) $\frac{\sqrt{33}}{7}$ (b) $\frac{\sqrt{33}}{4}$ (c) $\frac{7}{\sqrt{33}}$ (d) $\frac{4}{\sqrt{33}}$

Q11. The magnitude of the resultant of the vectors $\hat{i}+\hat{j}+\hat{k},\ 2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+2\hat{j}-\hat{k}$

(a) $\sqrt{39}$ (b) $\sqrt{40}$ (c) $\sqrt{41}$ (d) $\sqrt{42}$

Q12. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is :

(a) 1 (b) 2 (c) 3 (d) 4

Q13. Value of : $\int \sin^2 x\, dx$

(a) $\frac{1}{2}(x-\sin 2x)+c$ (b) $\frac{1}{2}(x+\sin 2x)+c$

(c) $\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)+c$ (d) $\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)+c$

Q14. If the direction cosines of a line are $\frac{1}{k},\frac{1}{k},\frac{1}{k}$ then the value of k is

(a) ± 1 (b) $\pm\frac{1}{\sqrt{3}}$ (c) $\pm\sqrt{3}$ (d) 0

Q15. Solve : $\int_0^4 |x-4|\, dx$

(a) 4 (b) 8 (c) 12 (d) 20

Q16. $f(x) = x(x-3)^2$ decreases for the values of x given by :

(a) $1 < x < 3$ (b) $-\infty < x < 1$

(c) $3 < x < \infty$ (d) None of these

Q17. If $\vec{a}\cdot\vec{b} = 8$ and $\vec{b} = 2\hat{i}+6\hat{j}+3\hat{k}$ then projection of $\vec{a}$ on $\vec{b}$ is

(a) $\frac{3}{7}$ (b) $\frac{8}{7}$ (c) $\frac{5}{7}$ (d) $\frac{1}{7}$

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Q18. If $A = \{1, 2, 3\}, B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y'. The range of R is

(a) $\{1, 4, 6, 9\}$ (b) $\{4, 6, 9\}$ (c) $\{1\}$ (d) None

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A) :** Given two sets $A = \{1, 2, 3\}, B = \{4, 5\}$ the number of injective function from A to B is zero.

Reason (R) : If $n(A) > n(B)$, then the number of injective functions from A to B is zero.

Q20. **Assertion (A) :** $f(x) = \sin^2 x$ is increasing function on $(0, \pi)$.

Reason (R) : A function $y = f(x)$ is said to be increasing on (a, b) if $f'(x) > 0 \ \forall \ x \in (a, b)$

SECTION B

Q21. Find the values of x and y : $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Q22 If $y = (\sin x)^{\tan x}$ find $\frac{dy}{dx}$.

Q23. Evaluate : $\int \frac{\sin x}{1 + \sin x} \, dx$

Q24. Solve the differential equations : $\frac{dy}{dx} = 1 + x + y + xy$.

Q25. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

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SECTION C

Q26. If $x\sin(a+y)+\sin a\cos(a+y)=0$, prove that $\dfrac{dy}{dx}=\dfrac{\sin^2(a+y)}{\sin a}$

Q27. A speaks truth in 75% and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in narrating the same incident ?

Q28. Evaluate : $\displaystyle\int_{-2}^{2}|1-x^2|\,dx$

OR

Evaluate : $\displaystyle\int_{-\pi/4}^{\pi/4}\log\left(\frac{1+\cot x}{1-\cot x}\right)dx$

Q29. Solve the differential equation : $\tan y\dfrac{dy}{dx}=\sin(x+y)+\sin(x-y)$

OR

Solve the differential equation : $(x^2+xy)\,dy=(x^2+y^2)\,dx$

Q30. Solve the following Linear Programming problems graphically :
Maximise $Z=4x+y$, Subject to the constraints :
$x+y\le 50$; $3x+y\le 90$; $x\ge 0, y\ge 0$

Q31. If $y=3\cos(\log x)+4\sin(\log x)$, , show that $x^2y_2+xy_1+y=0$.

SECTION D

Q32. Find the area bounded by the curve $y=|x-5|$, $y=0$, $x=2$ and $x=6$

Q33. The given relation R is defined on the set of real number as $a\,R\,b\Leftrightarrow|a|\le b.$ Find whether the given relation is reflexive, symmetric and transitive.

Q34. Find the foot of the perpendicular from the point (0, 2, 3) on the line $\dfrac{x+3}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}$.
Also, find the length of the perpendicular.

Q35. If $A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}$, show that $A^TA^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$

SECTION E

Q36. A cylindrical tank of fixed volume of $144\pi\, m^3$ is to be constructed with an open top to throw all the garbage in an orphanage. The manager of the orphanage called a contractor for the

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construction ensure that a tank to dispose off biodegradable waste can be constructed at a minimum cost.

(i) Find the cost of the least expensive tank that can be constructed if it costs Rs. 80 per sq. m for base and Rs. 120 per sq. m for walls.

(ii) Find the radius and height as well.

Q37. Read the following passage and the answer the questions given below.

Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ respectively.

(i) Write the direction ratios of the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$

(ii) Write a point, through which the line $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ passes.

(iii) Find the shortest distance between the given lines. Check if the lines intersect each other.

OR

(iii) Will the lines intersect each other? Find the point at which the motorcycles may collide

Q38. :Read the following passage and answer the questions given below.

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms, Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03. Assume that A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form.

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(i) Find the probability that Sonia processed the form and committed an error.

(ii) Find the probability that Vinay processed the form and committed an error.

(iii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is not processed by Vinay.

OR

(iii) Find the probability that Iqbal processed the form and committed an error. Find the total probability of committing an error in processing the form. Also, write the value of $\sum_{i=1}^{3} P(E_i | A)$.

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SAMPLE PAPER 2

SECTION A

Q1. If $R = \{(1,1),(2,2),(3,3),(4,a)\}$ be the identity relation on set A = {1, 2, 3, 4} then the value of a be

(a) $a = 1$ (b) $a = 2$ (c) $a = 3$ (d) $a = 4$

Q2. The matrices $\begin{bmatrix} 2x+y & 3x \\ 5x-y & 7y-4x \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 7 & 13 \end{bmatrix}$ if

(a) $x = 2, y = 5$ (b) $x = 3, y = 1$
(c) $x = 2, y = 3$ (d) $x = 3, y = 2$

Q3. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$

(a) 9 (b) –9 (c) 3 (d) –3

Q4. The domain of $f(x) = \sin^{-1} 3x$ is

(a) $\left[0, \frac{1}{3}\right]$ (b) $[-3, 3]$ (c) $[-1, 1]$ (d) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Q5. The function $f(x) = \dfrac{4 - x^2}{4x - x^3}$ is :

(a) discontinuous at only one point
(b) discontinuous at exactly two points
(c) discontinuous at exactly three points
(d) None of these

Q6. Exponential function $f(x) = e^x$ is

(a) always increasing (b) always decreasing
(c) both increasing and decreasing (d) neither increasing nor decreasing

Q7. If $y = \sin(x^x)$, then $\dfrac{dy}{dx}$ is :

(a) $x^x \cos(x^x)$ (b) $x^x \cos(x^x)(1 + \log x)$

(c) $x^x \cos(x^x) \log x$ (d) None of these

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Q8. The feasible region, for the constrains $x \geq 0, x+y \leq 5$ and $x-y \leq 5$ is situated in

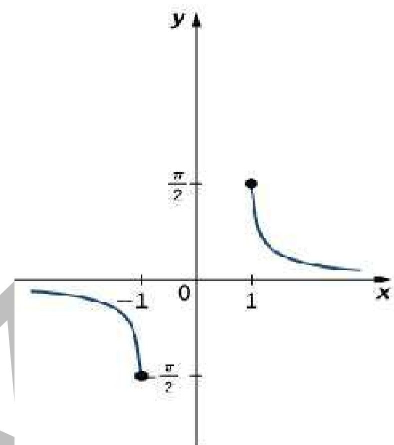
(a) I and II quadrant (b) II and III quadrant
(c) I and IV quadrant (d) I, II, III and IV quadrant

Q9. If $P(A)=\frac{3}{8}, P(B)=\frac{1}{2}, P(A \cap B)=\frac{1}{4}$, then $P(\bar{A}/\bar{B})$ is

(a) $\frac{5}{6}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) None

Q10. Identify the function shown in the graph :

(a) $\sin^{-1} x$ (b) $\operatorname{cosec}^{-1} x$
(c) $\sec^{-1} x$ (d) $\tan^{-1} x$



Q11. ABCD is a rhombus, whose diagonals intersect at E. Then, $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals to :

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BC}$ (d) $2\overrightarrow{AD}$

Q12. The order of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^3y}{dx^3}\right)$ is :

(a) 1 (b) 2 (c) 3 (d) Not defined

Q13. Value of $\int \cos 2x \cos 4x \, dx$

(a) $\frac{1}{2}\left\{\frac{\sin 6x}{6}-\frac{\sin 2x}{2}\right\}+c$ (b) $\frac{1}{2}\{\sin 6x+\sin 2x\}+c$

(c) $\frac{1}{2}\{\sin 6x-\sin 2x\}+c$ (d) $\frac{1}{2}\left\{\frac{\sin 6x}{6}+\frac{\sin 2x}{2}\right\}+c$

Q14. The angle between the lines $\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $-\frac{1-x}{1}=\frac{y+2}{2}=\frac{3-z}{-3}$ is :

(a) $\cos^{-1}\left|\frac{8}{\sqrt{70}}\right|$ (b) $\cos^{-1}\left|\frac{18}{\sqrt{70}}\right|$ (c) $\cos^{-1}\left|\frac{81}{\sqrt{70}}\right|$ (d) $\cos^{-1}\left|\frac{28}{\sqrt{70}}\right|$

Q15. Solve : $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x} dx$

MATHS KHA

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{4}$

Q16. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases, when side is 10 cm is :

(a) $10\,cm^2/s$ (b) $\sqrt{3}\,cm^2/s$ (c) $10\sqrt{3}\,cm^2/s$ (d) $\frac{10}{3}cm^2/s$

Q17. The value of λ such that the vectors $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ are orthogonal is :

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$

Q18. The range of real valued function $f(x)=\frac{x^2}{1+x^2}$

(a) R (b) $[0,\infty)$ (c) $(0,1)$ (d) $[0,1)$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A) :** A function $f:\mathrm{R}-\{5\}\to\mathrm{R}-\{2\}$ defined as $f(x)=\frac{2x+1}{x-5}$ is bijective.

Reason (R) : A function $f:\mathrm{A}\to\mathrm{B}$ is said to be surjective if $\forall\, y\in\mathrm{B}, \exists\, x\in\mathrm{A}$ such that $f(x)=y$.

Q20. **Assertion (A) :** The stationary point for $f(x)=x^x$ exists at $x=e$.

Reason (R) : For a function $y=f(x)$, the point $x=a$ where $f'(x)=0$ is called stationary point

SECTION B

Q21. Find X and Y if 2X + 3Y = $\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$.

Q22. Differentiation $y=\cos^{-1}\left(1-2^{2x+1}\right)$ w. r. t. x

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Q23. Evaluate : $\int \cos^5 x \, dx$

Q24. Solve the differential equations : $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$

Q25. Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points $(1,2,3)$ and $(4,5,6)$ respectively.

SECTION C

Q26. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, prove that $(1+x)^2 \frac{dy}{dx} + 1 = 0$.

Q27. A and B throw a coin alternately turn by turn.Whoever gets 'head' first wins. Find their respec tive chances of winning if A starts.

Q28. Solve : $\int_0^{2\pi} |\sin x| \, dx$ **OR** Solve : $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}} dx$

Q29. Solve the differential equations :Solve : $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$

OR

Solve the differential equations : $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$

Q30. Solve the following Linear Programming problems graphically :
Minimise $Z = 3x + 2y$, Subject to the costraints:
$x + y \geq 8$; $3x + 5y \leq 15$; $x \geq 0, y \geq 0$

Q31. If $y = \left(\sin^{-1} x\right)^2$, prove that $\left(1-x^2\right)y_2 - xy_1 - 2 = 0$.

SECTION D

Q32. Find the area of the region bounded by the following curves :
$y = 1 + |x+1|$; $x = -2$; $x = 3$ $y = 0$

Q33. Test whether the relation R on Z defined $R = \{(a,b) : |a-b| \leq 5\}$ is reflexive, symmetric and transitive.

Q34. Find the equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

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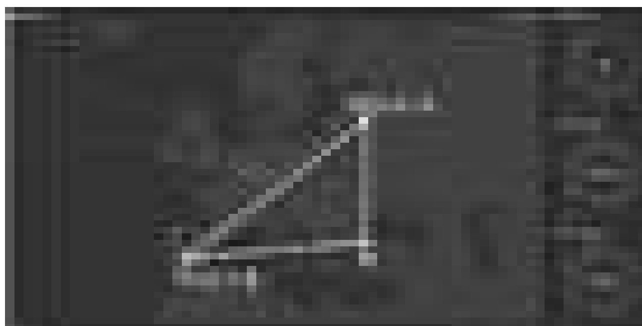
Q35. Solve the system of the equations : $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$.

SECTION E

Q36. The flight path of two airplanes in a flight simulator game are shown here. The coordinates of the airports $P(-2,1,3)$ and $Q(3,4,-1)$ are given .Airplane 1 flies directly from P to Q Airplane 2 has a layover at R and then flies to Q.The path of Airplane -2 from P to R can be represented by the vector $5\hat{i}+\hat{j}-2\hat{k}$

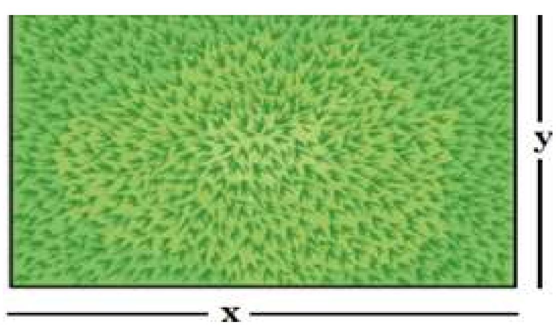
(i) Find the vector that represents the flight path of Airplane 1.
(ii) Find the vector representing the path of Airplane 2 from R to Q.
(iii) Find the angle between the flight paths of Airplane 1 and Airplane 2 just after take off?



OR

(iii) Consider that Airplane- 1 started the flight with a full fuel tank. Find the position vector of the point where one third of the fuel runs out if the entire fuel is required for the flight.

Q37. Read the following passage and the answer the questions given below.
Manjit wants to donate a rectangular plot of land for a school in his village.When he was asked to give dimensions of the plot, he told that :
* If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same,
* If length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2 .



(i) Assume that the length and breadth of the land be x and y (in metres) respectively. Find the equations in terms of x and y.
(ii) Using matrices, represent the linear equations obtained above in (i).
(iii) Using matrices, determine the dimensions of the land (in metres). Also write the area of the rectangular plot of land (in square metres).

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OR

(iii) Suppose that, Manjit gave the information about his plot in the following manner : If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain the same, but if length is decreased by 20 m and breadth is decreased by 10 m, then its area will be decreased by 4800 m^2 . In this situation, what will be dimensions of the plot? Assume that the length and breadth of the land be x and y (in metres) re spectively. Use matrices.

Q38. Read the following passage and answer the questions given below.
A mobile company in a town has 500 subscribers on its list and collects fixed charges of 300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs1/-, one subscriber will discontinue the service of this company.

(i) If the mobile company increases Rs x/-, then obtain the function R(x) , which represents the earning of the company. Also, find R `(x) .

(ii) What increase will bring maximum earning for the company? Use second derivative test.

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SAMPLE PAPER 3

SECTION A

Q1. If $A = \{a, b, c\}$ then the number of relations on A are

(a) 64 (b) 128 (c) 256 (d) 512

Q2. If the matrix A is both symmetric and skew symmetric then A is a

(a) diagonal matrix (b) zero matrix

(c) unit matrix (d) square matrix

Q3. If matrix $\begin{bmatrix} -1 & 2 \\ 4 & p \end{bmatrix}$ is singular, then value of p.

(a) 4 (b) 2 (c) –8 (d) 1

Q4. The domain of $f(x) = \cos^{-1} 5x$ is

(a) $[-5, 5]$ (b) $[0, 5]$ (c) $[-1, 1]$ (d) $\left[-\frac{1}{5}, \frac{1}{5}\right]$

Q5. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer function, is continuous at :

(a) 4 (b) –2 (c) 1 (d) 1.5

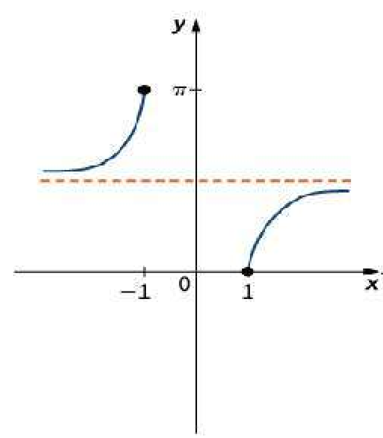
Q6. Which of the following function is decreasing on $\left(0, \frac{\pi}{2}\right)$:

(a) $\sin 2x$ (b) $\cos 3x$ (c) $\tan x$ (d) $\cos 2x$

Q7. Identify the function shown in the graph :

(a) $\tan^{-1} x$ (b) $\sec^{-1} x$

(c) $\cos^{-1} x$ (d) $\sin^{-1} x$



Q8. Which of the following statements is correct ?

(a) Every LPP has at least one optimal solution

(b) Every LPP has a unique optimal solution

(c) If an LPP has two optimal solutions, then it has infinitely many solution

(d) None of these

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Q9. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability of getting exactly one red ball is

(a) $\frac{45}{196}$ (b) $\frac{135}{392}$ (c) $\frac{15}{56}$ (d) None

Q10. The value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$ is

(a) $\frac{3\pi}{5}$ (b) $-\frac{7\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{10}$

Q11. If $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}+6\hat{j}-8\hat{k}$ are collinear the the value of 'a' is

(a) –2 (b) –3 (c) –4 (d) –5

Q12. Write the degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2 = x.\frac{dx}{dy}$.

(a) 1 (b) 2 (c) 3 (d) 4

Q13. Solve : $\int \frac{1}{\sqrt{x+3}-\sqrt{x+2}}\,dx$

(a) $\frac{2}{3}\left[(x+3)^{3/2}+(x+2)^{3/2}\right]+c$ (b) $\frac{2}{3}\left[(x+3)^{3/2}-(x+2)^{3/2}\right]+c$

(c) $\frac{3}{2}\left[(x+3)^{3/2}+(x+2)^{3/2}\right]+c$ (d) $\frac{3}{2}\left[(x+3)^{3/2}-(x+2)^{3/2}\right]+c$

Q14. The cartesian equation of a line is $2x+1=3y-2=4z+5,$ the direction ratio of the line defined by

(a) 2 : 4 : 5 (b) 2 : 3 : 4 (c) 3 : 4 : 6 (d) 6 : 4 : 3

Q15. Solve : $\int_0^{\pi/2} \log \tan x \, dx$

(a) 0 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Q16. The function $x^2-4x, x\in[0,4]$ attains minimum value at :

(a) $x=0$ (b) $x=2$ (c) $x=1$ (d) $x=4$

Q17. The position vectors of the vertices P, Q and R of ΔPQR are $-\hat{i}+2\hat{j}+4\hat{k}$, $3\hat{i}+6\hat{j}+8\hat{k}$ and $4\hat{i}+\hat{j}+\hat{k}$ respectively. Which of the following is the vector that represents the median $\overrightarrow{PS}$?

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(a) $\frac{7}{2}\hat{i}+\frac{9}{2}\hat{j}+\frac{9}{2}\hat{k}$ (b) $2\hat{i}+3\hat{j}+\frac{13}{3}\hat{k}$

(c) $\frac{9}{2}\hat{i}+\frac{3}{2}\hat{j}+\frac{1}{2}\hat{k}$ (d) $-\frac{1}{2}\hat{i}+\frac{5}{2}\hat{j}+\frac{7}{2}\hat{k}$

Q18. The domain of $f(x)=\sqrt{9-x^2}$ is

(a) $\mathrm{R}-\{\pm 3\}$ (b) $(-3,3)$ (c) $[-3,3]$ (d) R

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **<u>Assertion (A)</u> :** The signum function $f:\mathrm{R}\to\mathrm{R}$ given by $f(x)=\begin{cases}-1 & x<0\\ 0 & x=0\\ 1 & x>0\end{cases}$ is one-one

<u>Reason (R)</u> : A function $f:\mathrm{A}\to\mathrm{B}$ is said to be one-one if $f(a)=f(b) \Rightarrow a=b$ $\forall\, a,b\in\mathrm{A}$

Q20. **<u>Assertion (A)</u> :** The minimum value of $f(x)=3|x+1|+5$ is 5

<u>Reason (R)</u> : For A function $y=f(x)$ to be maximum/minimum in (a,b), it should have a stationary point in interval (a,b).

SECTION B

Q21. If $A=\begin{bmatrix}8 & 0\\ 4 & -2\\ 3 & 6\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2\\ 4 & 2\\ -5 & 1\end{bmatrix}$, then find the matrix X, such that $2A+3X=5B$.

Q22. If $y=x^{x^{x^{\dots\infty}}}$, find $\frac{dy}{dx}$.

Q23. Evaluate : $\int x\tan^{-1}x\,dx$

Q24. Solve the differential equations : $\sqrt{1+x^2}\,dy+\sqrt{1+y^2}\,dx=0$.

MATHS KHAZANA

Q25. Find the value of x for which $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector. Also calculate the vector along given vector having magnitude 7 units.

SECTION C

Q26. If $x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$, prove that $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$.

Q27. Rahul and Atul appear for an interview for two posts. The probabilities their selection are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. Find the probability that only one of them will be selected.

Q28. Solve : $\int_0^{\pi/2} |\cos x-\sin x|\ dx$ **OR** Solve : $\int_0^1 \log\left(\frac{1}{x}-1\right)dx$

Q29. Let a relation R on R be defined as $R=\{(a,b):1+ab>0; a,b\in R\}$. Show that R is reflexive, symmetric but not transitive .

Q30. Solve the linear programming probelm graphically :
Minimise $Z=4x+y$
Subject to the constraints : $x+y\le 50$, $3x+y\le 90$, $x\ge 0, y\ge 0$.

Q31. If $x=a\left(1-\cos\theta\right), y=a\left(\theta+\sin\theta\right)$, prove that : $\frac{d^2y}{dx^2}=-\frac{1}{a}$ at $\theta=\frac{\pi}{2}$.

SECTION D

Q32. Find the area bounded by $y=3-|x-1|,\ y=0,\ x=-2$ and $x=2$

Q33. Solve the differential equations : $\left(x^2-yx^2\right)dy+\left(y^2+x^2y^2\right)dx=0$.

OR

Solve the differential equations : $\left(x^2-y^2\right)dx+2xy\ dy=0$.

Q34. Find the direction cosines of the two lines which are connected by the relations $l-5m+3n=0$ and $7l^2+5m^2-3n^2=0$

Q35. If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the system of linear equations.

$x+2y+z=4$, $-x+y+z=0$, $x-3y+z=2$

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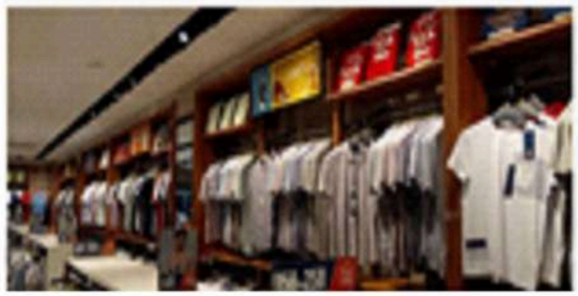
SECTION E

Q36. Read the following passage and answer the questions given below.

Rahul went to a nearby market for shopping of some readymade garments. The probability that Rahul will buy a shirt is 0.2, the probability that he will buy a trouser is 0.3, and the probabil ity that he will buy a shirt given that he buys a trouser is 0.4.



(i) Find the probability that Rahul will buy both a shirt and a trouser.

(ii) Find the probability that Rahul will buy a trouser given that he buys a shirt.

Q37. Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs 25, Rs. 100 and Rs 50 each respectively. The numbers of articles sold are given as

Based on the information given above, answer the following questions :

School / Articles	DPS	CVC	KVS
Handmade fans	40	25	30
Mats	50	40	50
Plates	20	30	40

(i) What is the total money (in Rupees) collected by the school DPS ?

(ii) What is the total amount of money collected by all three schools DPS, CVC and KVS ?

(iii) If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools ?

Q38. An open box is to be made out of a piece of a cardboard measuring $24\,cm \times 24\,cm$, by cutting of equal squares of x cm from the corners and turning up the sides.

(i) What is the volume of the box in terms of x ?

(ii) Find the rate of change of volume with respect to x

(iii) What will be the value of x for maximum volume.

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SAMPLE PAPER 4

SCETION A

Q1. Which of the following can't be a function ?

(a) $R = \{(1,2)\ (2,3)(4,5)\}$ (b) $R = \{(1,2)\ (4,6)(0,7)\}$

(c) $R = \{(1,2)\ (4,6)(1,5)(2,9)\}$ (d) $R = \{(0,1)\ (1,2)(2,3)\}$

Q2. If matrix $A = [a_{ij}]_{2\times 2}$ where $a_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$, then A^2 is equal to

(a) I (b) 3 A (c) O (d) None of these

Q3. Given that A is a non-singular matrix of order 3, such that $A^2 = 2A$, then value of |2A| is :

(a) 4 (b) 8 (c) 64 (d) 16

Q4. The domain of $f(x) = \sin^{-1}(1-5x)$ is

(a) $\left[-\frac{1}{5}, \frac{1}{5}\right]$ (b) $[-5, 0]$ (c) $\left[0, \frac{2}{5}\right]$ (d) None

Q5. The value of the constant *k* so that the function $f(x)$ is continuous at $x = 0$ where

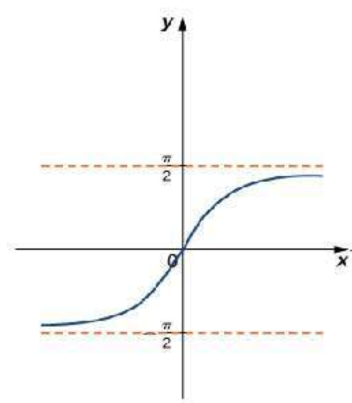
$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases} \text{ is :}$$

(a) 1 (b) 0 (c) -1 (d) None of these

Q6. Identify the function shown in the graph :

(a) $\sin^{-1} x$ (b) $\cos^{-1} x$

(c) $\sec^{-1} x$ (d) $\tan^{-1} x$



Q7. If $y = \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$, then $\frac{dy}{dx}$ is :

(a) $\frac{1}{1+x}$ (b) $\frac{1}{\sqrt{x}(1+x)}$ (c) $\frac{2}{\sqrt{x}(1+x)}$ (d) $\frac{1}{2\sqrt{x}(1+x)}$

Q8. The corner points of the feasible region determined by the system of linear constraints are $(0,10),(5,5),(15,15),(0,20)$. Let $Z = px + qy,$ where $p, q > 0.$ Condition on *p* an *q* so that

MATHS KHAZANA

the maximum of Z occurs at the points $(15,15)$ and $(0,20)$ both, is

(a) $p=q$ (b) $p=2q$ (c) $q=2p$ (d) $q=3p$

Q9. If the projection of $\lambda\hat{i}+\hat{j}+4\hat{k}$ on $2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units, then the value of λ is equal to :

(a) 3 (b) 5 (c) -7 (d) -9

Q10. If $\operatorname{cosec}^{-1}\frac{a}{x}=\sec^{-1}\frac{a}{y}$ then $\sin^{-1}\frac{x}{a}+\sin^{-1}\frac{y}{a}$ is equal to

(a) π (b) $\frac{3\pi}{2}$ (c) $\frac{\pi}{2}$ (d) 0

Q11. Given two independent events A and B such that P (A) = 0.3, P (B) = 0.6 and $P(A'\cap B')$ is :

(a) 0.9 (b) 0.18 (c) 0.28 (d) 0.1

Q12. Write the degree of the differential equation $\frac{d^2y}{dx^2}+\sqrt{x}=\left(\frac{dy}{dx}\right)^{1/3}$.

(a) 1 (b) 2 (c) 3 (d) 4

Q13. Value of $\int\frac{\sin(x-a)}{\sin x}\,dx$

(a) $x\sin a-\sin a\log|\sin x|+c$ (b) $x\cos a-\sin a\log|\sin x|+c$

(c) $x\cos a+\sin a\log|\sin x|+c$ (d) $x\sin a+\sin a\log|\sin x|+c$

Q14. Cartesian equation of the line represented by $\vec{r}=2\hat{i}+\hat{j}-4\hat{k}+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$ is :

(a) $\frac{x-2}{1}=\frac{y+1}{-1}=\frac{z+4}{-1}=\lambda$ (b) $\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+4}{-1}=\lambda$

(c) $\frac{x-2}{1}=\frac{y+1}{-1}=\frac{z-4}{-1}=\lambda$ (d) $\frac{x-2}{1}=\frac{y-1}{1}=\frac{z+4}{-1}=\lambda$

Q15 Solve : $\int e^x\frac{x-1}{(x+1)^3}\,dx$

(a) $\frac{e^x}{(x+1)^3}+c$ (b) $\frac{e^x}{(1+x)^2}+c$ (c) $-\frac{e^x}{(x+1)^3}+c$ (d) $-\frac{e^x}{(x+1)^2}+c$

Q16. For all real values of x, increasing function is :

(a) $f(x)=\frac{1}{x}$ (b) $f(x)=x^2$ (c) $f(x)=x^3$ (d) $f(x)=x^4$

Q17. If A,B,C,D are the points with position vectors $\hat{i}+\hat{j}-\hat{k}$, $2\hat{i}-\hat{j}+3\hat{k}$, $2\hat{i}-3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$ respectively, then the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ is

(a) $\sqrt{12}$ (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{21}$

MATHS KHAZANA

Q18. If $\cos\left(\sin^{-1}\frac{2}{5}+\cos^{-1}x\right)=0,$ then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) 0 (d) 1

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A)** : The modulus function $f: Z-\{0\}\to \mathrm{N}$ given by $f(x)=|x|$ is surjective

Reason (R) : A function $f:\mathrm{A}\to\mathrm{B}$ is said to be surjective when for every elements of set B there exists a pre-images in set A.

Q20. **Assertion (A) :** The maximum value of $f(x)=\sin x+\cos x$ is 1

Reason (R) : The maximum value of $\sin x$ is 1

SECTION B

Q21. If A = $\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and I = $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k if $A^2 = kA - 2I$.

Q22. Find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{3}$ if $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$

Q23. Evaluate $\int e^x \sin x\, dx$

Q24. Solve : $\sec^2 x\tan y\, dx+\sec^2 y\tan x\, dy=0$

Q25. If $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}-3\hat{k},$ find $\left|\vec{a}-2\vec{b}\right|$.

SECTION C

Q26 . If $x^m y^n=(x+y)^{m+n}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

MATHS KHAZANA

Q27. A, B and C throw a die turn by turn starting with A. Whoever throws 'six' first wins. Find their respective chances of winning.

Q28. Solve : $\int_0^{\pi/4} \log(1+\tan x)\, dx$ **OR** $\int_0^{\pi} \frac{x \tan x}{\tan x + \sec x}\, dx$

Q29. Solve the differential equation : $(e^y + 1)\cos x\ dx + e^y \sin x\ dy = 0$

OR

Solve the differential equation : $\left(1+e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right) dy = 0$

Q30. Solve the following Linear Programming problems graphically :
Minimise Z = 200 x + 500 y, Subject to the constraints :
$x + 2y \geq 10$; $3x + 4y \leq 24$; $x \geq 0, y \geq 0$;

Q31. $y = x^x$, prove that : $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0.$

OR

If $(x - a)^2 + (y - b)^2 = c^2$, for some $x > 0$, prove that : $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

SECTION D

Q32. Find the area bounded by the curve $y^2 = 4ax$ and the lines $y = 2a$ and y- axis.

Q33. Prove that relation R on Z defined by $(a, b) \in R \Leftrightarrow a - b$ is divisible by 5, is an equivalence relation on Z.

Q34. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Q35. Solve the system of equations by matrix method.
$3x - 2y + 3z = 8$, $2x + y - z = 1$, $4x - 3y + 2z = 4$

MATHS KHAZANA

SECTION E

Q36. Read the following passage and answer the questions given below.
A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uni - form rate of $2\,cm^3/s$. The semi-vertical angle of the conical tank is $45°$.

(i) Find the volume of water in the tank in terms of its radius *r*.

(ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm.

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm.

OR

(iii) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

Q37. The equation of motion of a missile are x = 3t, y = -4t, z = t, where the time 't' is given inseconds, and the distance is measured in kilometres.

Based on the above answer the following:

(i). At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

(ii). If the position of rocket at a certain instant of time is $(5, -8, 10)$, then what will be theheight of the rocket from the ground? (The ground is considered as the xy – plane).

MATHS KHAZANA

Q38. There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?

MATHS KHAZANA

SAMPLE PAPER 5

SECTION A

Q1. Let $n(\mathrm{A}) = 4$ $n(\mathrm{B}) = 3$ the number of functions from A into B

(a) 9 (b) 27 (c) 81 (d) None

Q2. If the matrix $\mathrm{A} = \begin{bmatrix} 5 & 2 & x \\ y & 5 & 6 \\ 7 & 6 & x \end{bmatrix}$ is symmetric, the values of x^y is :

(a) 2 (b) 7 (c) 49 (d) 128

Q3. If $\mathrm{A} = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} does not exists for

(a) $\lambda = 2$ (b) $\lambda = -\frac{8}{5}$

(c) $\lambda = -8$ (d) None of these

Q4. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to :

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

Q5. The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ continuous at $x = 0$, then the value of k is :

(a) 3 (b) 2 (c) 1 (d) 1.5

Q6. The minimum value of $\sin x + \cos x$ is :

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) -2 (d) None

Q7. The value of $\frac{d}{dx}\left(\sin^{-1}\frac{x}{3} + \cos^{-1}\frac{x}{3}\right)$ is equal to :

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) None

MATHS KHAZANA

Q8. In defining of an LPP, if m stands for the number of constraints and n for the number of variables then which of the following is true ?

(a) $m = n$ (b) $m \geq n$ (c) $m \leq n$ (d) None

Q9. A card is picked at random from a pack of 52 playing cards.Given that the picked card is a queen , the probability of this card to be a card of spade is

(a) $\frac{1}{3}$ (b) $\frac{4}{13}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Q10. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $xy + yz + zx$ is

(a) 1 (b) 3 (c) 2 (d) 5

Q11. If $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$ for a unit vector $\vec{a}$ then $|\vec{x}|$ is

(a) 2 units (b) 3 units (c) 4 units (d) 5 units

Q12. Write the degree of the differential equation $\left[1 + \left(\frac{d^2y}{dx^2}\right)^2\right]^{3/2} = \frac{dy}{dx}$.

(a) 3/2 (b) 2 (c) 3 (d) 6

Q13. Solve : $\int \frac{x+3}{x-1} dx$

(a) $x + 4\log|x+1| + c$ (b) $x - 4\log|x-1| + c$

(c) $x + 4\log|x-1| + c$ (d) $4x - \log|x-1| + c$

Q14. If a line makes angles $90°$ and $60°$ respectively with x and y axes then the acute angle made by the line with z - axis is

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

Q15. Solve : $\int_0^{\pi} |\cos x| \, dx$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

Q16. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$ has :

(a) two points of local maximum (b) two points of local minimum

(c) one maxima and one minima (d) no maxima or minima

Q17. The value of $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$ is

MATHS KHAZANA

(a) 0 (b) -1 (c) 1 (d) 3

Q18. If A and B are independent events and $P(A \cup B) = \frac{3}{8}$, then $P(A') \times P(B')$ is

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) $\frac{7}{8}$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **<u>Assertion (A)</u> :** The constant function $f : R \to R, f(x) = 5$ is injective.

<u>Reason (R)</u> : A function $f : A \to B$ is surjective if every element of set B has a pre-image in set A.

Q20. **<u>Assertion (A)</u> :** A function $f(x) = \tan x - x$ is an increasing function on its domain

<u>Reason (R)</u> : A function $y = f(x)$ is said to be increasing on (a,b) if $f'(x) > 0 \ \forall\ x \in (a,b)$

SECTION B

Q21. If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = O$, find x.

Q22. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \text{ to } \infty}}}$, prove that $\frac{dy}{dx} = \frac{\cos x}{2y - 1}$

Q23. Evaluate : $\int \frac{1}{5 + 3\sin^2 x} dx$

Q24. Solve : $\frac{dy}{dx} + 3y = e^{-2x}$

Q25. If $\vec{a}$ is any vector , prove that $\vec{a} = (\vec{a}.\hat{i}).\hat{i} + (\vec{a}.\hat{j}).\hat{j} + (\vec{a}.\hat{k}).\hat{k}$

MATHS KHAZANA

SECTION C

Q26. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

Q27. The odds against A solving a problem are 4 to 3 and the odds in favour of B solving it are 7 to 5.Find the probability that the problem is solved if they try independently.

Q28. Solve : $\int_0^{\pi/4}\log(1+\tan x)\,dx$

OR

Solve : $\int_{-\pi/4}^{\pi/4}|\sin x|\,dx$

Q29. Solve the differential equations : $(1+y^2)(1+\log x)\,dx+x\,dy=0$ given that when $x=1$, $y=1$.

OR

Solve the differential equations : $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

Q30. Solve the following Linear Programming problems graphically :
Minimise and Maximise $Z = 3x + 9y$, Subject to the constraints
$x + 3y \le 60$; $x + y \ge 10$; $x \le y$; $x \ge 0, y \ge 0$

Q31. If $y=e^x(\sin x+\cos x)$ prove that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0.$

SECTION D

Q32. Using integration find the area of region bounded by the triangle whose vertices are $(-1,0),(1,3)$ and $(3,2)$.

Q33. Prove that the relation R on set $N\times N$ defined by. $(a,b)R(c,d)\Leftrightarrow a+d=b+c$, for all $(a,b),(c,d)\in N\times N$ is an equivalence relation.

Q34. Show that the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z+6}{-5}$ intersect. Find their point of intersection.

Q35. Express the matrix $A=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

SECTION E

Q36. Read the following passage and answer the questions given below. Utkarsh was doing a survey on a school. Theme of the survey was 'the average number of

MATHS KHAZANA

hours spent on study' by students selected at random. At the end of survey, he prepared the following report related to the data. Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some constant.

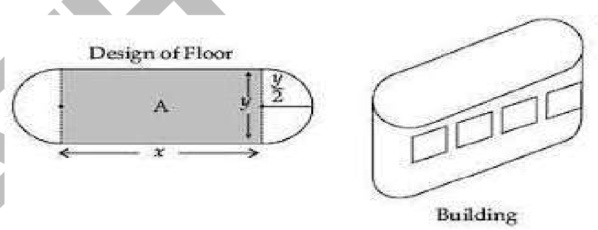
$$P(X=x)=\begin{cases} 0.2, \text{ if } x=0 \\ kx, \text{ if } x=1 \text{ or } 2 \\ k(6-x), \text{ if } x=3 \text{ or } 4 \\ 0, \text{ otherwise} \end{cases}$$

(i) What is the value of k?
(ii) What is the probability that the average study time of students is not more than 1 hour?
(iii) What is the probability that the average study time to students is at least 3 hours?
What is the probability that the average study time of students is exactly 2 hours?

OR

(iii) What is the probability that the average study time of students is at least 1 hour?

Q37. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below:



Based on the above information answer the following:
(A) If x and y represent the length and breadth of the rectangular region, then establish the relation between the variables .
(B) Explain the area of the rectangular region A in terms x.
(C) What is the maximum value of area A ?

Q38. An amount of Rs 10,000 is put into three Bonds at the rate of 10, 12 and 15% per *annum.* The combined incomes is Rs 1,310 and the combined income of the first and second Bond is Rs 190 short of the income from the third.
(i) Express the given problem in terms of x, y and z
(ii) Find the inverse of coefficient matrix
(iii) Find the investments in each bond.

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SAMPLE PAPER 6

SECTION - A

Q1. $f : \mathrm{R} \to \mathrm{R} : f(x) = 2x$ is

(a) One-one and onto (b) One-one and into

(c) Many-one and onto (d) Many-one and into

Q2. If A, B and C are matrices of orders $m \times n$, $3 \times p$ and $q \times 3$ respectively such that the matrix . $\mathrm{A}(\mathrm{B}+\mathrm{C})$ has 6 elements then the value of m.

(a) 1 (b) 2 (c) 3 (d) None

Q3. If $\mathrm{A} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|\, adj\ \mathrm{A}\,|$ is :

(a) a^{27} (b) a^6 (c) a^9 (d) a^2

Q4. If $\tan^{-1} x = \dfrac{\pi}{8}$ for some $x \in R$, then the value of $\cot^{-1} x$ is

(a) $\dfrac{\pi}{8}$ (b) $\dfrac{2\pi}{5}$ (c) $\dfrac{3\pi}{8}$ (d) $\dfrac{4\pi}{5}$

Q5. The points of discontinuity for the function $f(x) = [x]$ in $-3 < x < 3$ are at :

(a) 0 (b) ± 1 (c) ± 2 (d) all are correct

Q6. The interval on which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing is :

(a) $(-1, \infty)$ (b) $(-2, -1)$

(c) $(-\infty, -2)$ (d) $(-1, 1)$

Q7. The derivative of $\cos^{-1}(2x^2 - 1)$ w. r. t. $\cos^{-1} x$ is :

(a) 2 (b) $\dfrac{-1}{2\sqrt{1-x^2}}$ (c) $\dfrac{2}{x}$ (d) $1 - x^2$

Q8. Corner points of the feasible region determined by the system of linear constraints are $(0,3), (1,1)$ and $(3,0)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p an q so that the maximum of Z occurs at $(3,0)$ and $(1,1)$ is

(a) $p = 2q$ (b) $p = \dfrac{q}{2}$ (c) $p = 3q$ (d) $p = q$

MATHS KHAZANA

Q9. Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

Q10. The domain of $f(x)=\sin^{-1}(2x-1)+\cos^{-1}3x$

(a) $\left[-\frac{1}{3},\frac{1}{3}\right]$ (b) $[0,1]$ (c) $\left[0,\frac{1}{3}\right]$ (d) $[-1,1]$

Q11. Unit vector perpendicular to both $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$ is

(a) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (b) $\frac{-\hat{i}+\hat{j}}{\sqrt{2}}$ (c) $\frac{-\hat{i}-\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

Q12. The order and degree of the differential equation $2\left(\frac{d^2y}{dx^2}\right)=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}$ respectively are :

(a) 1, 2 (b) 2, 1 (c) 2, 2 (d) 2, 3

Q13. Value of : $\int 2^{x+2}\, e^{x-1}\, dx$

(a) $\frac{2^{x+2}\, e^{x-1}}{\log 2}+c$ (b) $\frac{4e(2e)^x}{\log(2e)}+c$ (c) $\frac{4}{e}\frac{(2e)^x}{\log(2e)}+c$ (d) $\frac{(2e)^x}{\log(2e)}+c$

Q14. Vector equation of line passing through $(1,2,-1)$ and parallel to the $x-25=6-2y=7-z$ is :

(a) $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(2\hat{i}-\hat{j}-2\hat{k})$ (b) $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+\lambda(2\hat{i}+\hat{j}-2\hat{k})$

(c) $\vec{r}=\hat{i}-2\hat{j}-\hat{k}+\lambda(2\hat{i}-\hat{j}+2\hat{k})$ (d) $\vec{r}=\hat{i}-2\hat{j}+\hat{k}+\lambda(2\hat{i}-\hat{j}+2\hat{k})$

Q15. Solve : $\int_0^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\, dx$

(a) 2 (b) 5 (c) $\frac{5}{2}$ (d) 10

Q16. The function $f(x)=\frac{x}{3}+\frac{3}{x}, x\neq 0$ increases on :

(a) $(-3,0)$ (b) $(0,3)$ (c) $(-3,3)$ (d) None

Q17. Two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. The length of the median through A is

MATHS KHAZANA

(a) $\frac{\sqrt{34}}{2}$ (b) $2\sqrt{6}$ (c) $\frac{3\sqrt{2}}{2}$ (d) None of these

Q18. If $f(x)=\sec^{-1}x+\tan^{-1}x,$ then $f(x)$ is real for

(a) $x\in(-\infty,-1]\cup[1,\infty)$ (b) $(-1,1)$

(c) $x\in[1,1]$ (d) None

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **<u>Assertion (A)</u> :** Domain of function $f(x)=\cos^{-1}(3x-1)$ is $\left[0,\frac{2}{3}\right]$

<u>Reason (R)</u> : Domain of $\cos^{-1}x$ is $[0,\pi]$

Q20. **<u>Assertion (A)</u> :** If $y=\sin x°$ then $\frac{dy}{dx}=\frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right)^{\circ}$

<u>Reason (R)</u> : $\frac{d}{dx}\left(\sin^{-1}\right)=\frac{1}{\sqrt{1-x^2}}$

SECTION - B

Q21. Given $3\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the value of *x, y, z and w.*

Q22. If $y=(\sin x)^{\tan x}$ find $\frac{dy}{dx}$.

Q23. Solve : $\int\frac{\sin 2x}{\sin^2 x+5}\,dx$

Q24. Solve the differential equations : $\frac{dy}{dx}+2y=\sin x$

Q25. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}\times\vec{b}|=\vec{a}.\vec{b},$ then what is the angle between $\vec{a}$ and $\vec{b}$

MATHS KHAZANA

SECTION - C

Q26. If $y = \sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$, find $\frac{dy}{dx}$.

Q27. A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that the problem is solved?

Q28. Solve $\int_0^{\pi/2} \log \sin x \; dx$ **OR** Solve : $\int_{-\pi/2}^{\pi/2} \frac{dx}{e^{\sin x}+1}$

Q29. Solve differential equation : $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$

OR

Find the equation of a curve passing through the point $(0, -2)$ given that at any point (x,y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.

Q30. Solve the following Linear Programming Problems graphically :
Maximise $Z = x + y$, subject to $x - y \le -1, -x + y \le 0, x, y \ge 0$.

Q31. Prove that : $\frac{d}{dx}\left[\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x+\sqrt{x^2+a^2})\right] = \sqrt{x^2+a^2}$

OR

Find the value of k for which the function f defined by $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x < \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \\ \frac{3\tan 2x}{2x-\pi}, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

SECTION - D

Q32. Find the area of the region lying in the first quadrant and bounded by $y = 4x^2, x = 0, y = 1$ and $y = 4$.

Q33. Let A = R − {3} and B = R − {1}. Consider the function f : A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto ? Justify your answer.

MATHS KHAZANA

Q34. Find the foot of perpendicular from the point (2, 3, 4) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

Q35. Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations :

$x+3z=9,\ -x+2y-2z=4,\ 2x-3y+4z=-3$

OR

Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations :

$x-y+2z=1;\quad 2y-3z=1;\quad 3x-2y+4z=2$

MATHS KHAZANA

SECTION - E

Q36. The Indian coast guard, while patrolling, saw a suspicious boat with people.The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3,5) and (2, 5, 3) respectively

Based on the above answer the following:

1. If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in meters) that the bullet has to travel?

2. If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36m/sec, then what is the time taken for the bullet to travel and hit the boat?

3. At that given instant of time, what is the equation of line passing through the positions of the helicopter and boat ?

4. At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat if the coast guard

shoots the bullet along the line whose equation is $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-5}{-2}$ for the bullet to hit the boat?

Q37. An insurance company insured 5000 truck drivers, 3000 car drivers and 2000 scooter drivers. The chance of an accident of truck driver is 0.6, that of car driver is 0.4 and that of scooter is 0.2. Let events A, B and C represents insured truck drivers, car drivers and scooter drivers and event E represents that an insured person meets an accident. Based on above information answer the following questions

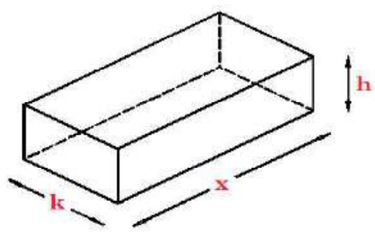
(i) What is the probability that an insured person meets with an accident ?

(ii) What is the probability that an insured person meets with an accident given that the person is car driver ?

(iii) What is the probability that insured person is car driver given that the insured person meets with an accident ?

Q38. Read the following passage and then answer the questions given below.

A foreign client approaches VERMA BRICKS COMPANY for a special type of bricks.
The client requests for few samples of bricks as per their requirement.
The solid rectangular brick is to be made from 1 cubic feet of clay of special type.
The brick must be 3 times as long as it is wide.



(i) According to the figure shown, the length of brick is 'x', width is 'k' and height is 'h'. Obtain an expression in terms of 'h' and 'k'.

(ii) Express the surface area (S) of the brick, as a function of 'k'.

(iii) Find $\frac{dS}{dk}$. At what value of k, $\frac{dS}{dk}=0$?

Show that $\frac{d^2S}{dk^2}$ is positive, at this obtained value of k. What does it signify?

OR

(iii) Find the minimum value of S, using second derivative test.

MATHS KHAZANA

SAMPLE PAPER 7

SECTION A

Q1. Let $'f': R-\{2\} \to R-\{1\}$ be a function defined by $f(x)=\frac{x-1}{x-2}$, then $'f'$ is

(a) One-one and into function (b) Many-one and onto function
(c) Bijective function (d) Many one and into function

Q2. The number of all possible matrices of order 2×3 with each element either 4 or 5 or 6 is

(a) 2^3 (b) 6^3 (c) 3^6 (d) 2^6

Q3. If $A=\begin{pmatrix} x & 0 \\ 2 & x \end{pmatrix}$, and $|A^3|=64$, then value of x is :

(a) ± 1 (b) ± 3 (c) ± 2 (d) ± 5

Q4. The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is

(a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) None

Q5. If A is square matrix such that $A^2-A+I=O$ then A^{-1} will be

(a) $A-I$ (b) $I+A$ (c) A (d) $I-A$

Q6. $f(x)=x^x$ has a stationary point at :

(a) $x=e$ (b) $x=\frac{1}{e}$ (c) $x=1$ (d) $x=\sqrt{e}$

Q7. If $f(x) = 2x$ and $g(x) = \frac{x^2}{2}+1$, then which of the following can be a discontinuous function :

(a) $f(x)+g(x)$ (b) $f(x)-g(x)$

(c) $f(x)\ g(x)$ (d) $\frac{g(x)}{f(x)}$

Q8. The graph of the inequality $2x+3y>6$ is

(a) half plane that contains the origin
(b) half plane that neither contains the origin nor the points of the line $2x+3y=6$.
(c) whole XOY - plane excluding the points on line $2x+3y=6$.
(d) entire XOY-plane.

MATHS KHAZANA

Q9. A and B are events such that $P(A/B)=P(B/A)$ then

(a) $A\subset B$ (b) $B=A$ (c) $A\cap B=\phi$ (d) $P(A)=P(B)$

Q10. The domain of $f(x)=\sin^{-1}x+\sec^{-1}(2x)$

(a) $[-1,1]$ (b) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (c) $\left[-1,-\frac{1}{2}\right]\cup\left[\frac{1}{2},1\right]$ (d) None

Q11. The value of λ which makes $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+3\hat{k}$ perpendicular is

(a) $\frac{2}{5}$ (b) $-\frac{2}{5}$ (c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

Q12. If the slope of tangent to curve $y=f(x)$ at any point (x,y) is the product of abcissa and sum of abcissa and ordinate then differential equation is

(a) $\frac{dy}{dx}=xy$ (b) $\frac{dy}{dx}=x+y$ (c) $\frac{dy}{dx}=x(x+y)$ (d) $\frac{dy}{dx}=y(x+y)$

Q13. Value of : $\int\frac{e^x}{e^x+5}\,dx$

(a) $\log\left|e^x+5\right|+c$ (b) $x\log\left|e^x+5\right|+c$

(c) $e^x\log\left|e^x+5\right|+c$ (d) $\frac{e^x}{\log\left|e^x+5\right|}+c$

Q14. The angle between lines $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\hat{i}-\hat{j}+\hat{k}+\mu\left(2\hat{i}-\hat{j}+\hat{k}\right)$ is :

(a) $\theta=\cos^{-1}\left|\frac{11}{7\sqrt{6}}\right|$ (b) $\theta=\cos^{-1}\left|\frac{10}{7\sqrt{6}}\right|$ (c) $\theta=\cos^{-1}\left|\frac{12}{7\sqrt{6}}\right|$ (d) $\theta=\cos^{-1}\left|\frac{9}{7\sqrt{6}}\right|$

Q15. Value of : $\int\frac{\sqrt{5-\sqrt{x}}}{\sqrt{x}}\,dx$

(a) $\frac{4}{3}\left(5-\sqrt{x}\right)^{3/2}+c$ (b) $-\frac{3}{4}\left(5-\sqrt{x}\right)^{3/2}+c$

(c) $-\frac{4}{3}\left(5-\sqrt{x}\right)^{3/2}+c$ (d) $-\frac{8}{3}\left(5-\sqrt{x}\right)^{3/2}+c$

Q16. The maximum value of $f(x)=\frac{\log x}{x}$, if it exists, is :

MATHS KHAZANA

(a) e (b) $-\frac{1}{e}$ (c) $\frac{1}{e}$ (d) None

Q17. The unit vector perpendicular to vectors $\hat{i}-\hat{j}$ and $\hat{i}+\hat{j}$ forming a right-handed system is

(a) $\hat{k}$ (b) $-\frac{\hat{k}}{2}$ (c) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ (d) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Q18. If $\tan^{-1}x+\tan^{-1}y=\frac{4\pi}{5}$, then $\cot^{-1}x+\cot^{-1}y$ equals

(a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) π

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A)** : If $\cos^{-1}x+3\sin^{-1}x=\pi$ then $x=\frac{1}{\sqrt{2}}$.

Reason (R) : $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$ and $\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Q20. **Assertion (A)** : If $y=\sin\theta$ $x=\cos\theta$ then $\frac{dy}{dx}=-\cot\theta$.

Reason (R) : If a function is defined as $y=f(\theta)$ $x=g(\theta)$ then $\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{f'(\theta)}{g'(\theta)}$

SECTION B

Q21. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find k such that $A^2-8A+kI=0$.

Q22. Differentiate w.r.t. x : $y=\log_7(\log x)$

Q23. Solve : $\int\frac{5x-3}{(x-1)^4}\,dx$

MATHS KHAZANA

Q24. Solve the differential equations : $\frac{dy}{dx} = 1 - x + y - xy$

Q25. Find $\left(\vec{a} + 3\vec{b}\right) \cdot \left(2\vec{a} - \vec{b}\right)$, if $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$.

SECTION C

Q26. Differentiate w.r.t. x : $\tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$

Q27. A and B are playing a game. A throws a die and B tosses a coin turn by turn. A wins the game if he gets a number more than 4 on the die and B wins if he gets a head. Find their respective chances of winning if A starts the game.

Q28. Evaluate: $\int_{-1}^{2} |x^3 - x|\, dx$ **OR** Evaluate : $\int_{0}^{1} x(1-x)^n\, dx$

Q29. Solve the differential equation : $y\, dx + x \log\left(\frac{y}{x}\right) dy - 2x\, dy = 0$

OR

Find the equation of a curve passing through the point $(-2, 3)$, given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

Q30. Solve the following Linear Programming Problems graphically :
Minimise $Z = x + 2y$ subject to $2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Q31. If $y = \sin(\log x)$, Prove that $x^2 \frac{d^2 y}{dx} + x\frac{dy}{dx} + y = 0$

OR

Find the value of a when the function is continuous at $x = 0$. $f(x) = \begin{cases} a\sin\frac{\pi}{2}(x+1) & ,x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & ,x > 0 \end{cases}$

SECTION D

Q32. Find the area bounded by $5x - 2y + 10 = 0$, $x + y = 5$ and $y = 0$

MATHS KHAZANA

Q33. Prove that the function $f : \mathbf{N} \to \mathbf{N}$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

Q34. Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the joint of the points B (4, 7, 1) and C (3, 5, 3)

Q35. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equation :

$x - y + z = 4$, $\quad x - 2y - 2z = 9$, $\quad 2x + y + 3z = 1$

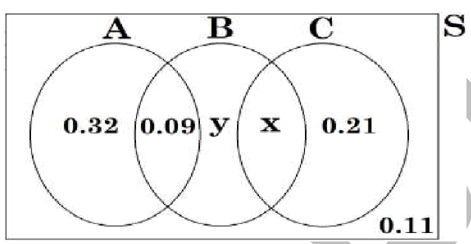
SECTION - E

Q36. Read the following passage and then answer the questions given below.

There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below:

The venn diagram below represents the probabilities of three different types of Yoga A, B and C performed by the people of a society.

Further, it is given that probability of a member performing type C Yoga is 0.44.



(i) Find the value of x.

(ii) Find the value of y.

(iii) Find $P(A|B)$ and $P(C|B)$.

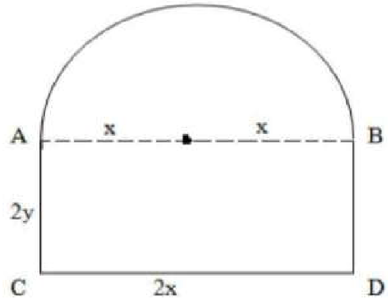
OR

(iii) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.

Q37. Mr.Rahul who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semicircular opening. This window is having a perimeter of 10m as shown below :

Based on the above information answer the following questions.

MATHS KHAZANA



(i) If $2x$ and $2y$ are the length and breadth of the rectangular portion of the window, then find the relation between variables

(ii) Find the combined area A of rectangular region and semi-circular region of the widow in terms of x.

(iii) Find the maximum value of area A of the whole window.

Q38. Two schools P and Q want to award their students on the basis of Honesty, Discipline and Social work done by them during Covid pandemic. School P announces Rs x each, Rs y each and Rs z each for three respective values to its 2, 3 and 4 students respectively with a total award money of Rs 6,100. School Q announces total amount Rs 6,300 as award money to its 1, 2 and 5 students on the respective values. One student received prize on each value amounting Rs 1,800 Using the information answer the following :

(i) Express the given problem as linear equations.

(ii) Find the award money for the value Discipline.

(iii) Find the award money for the value Social work.

OR

If A and B are invertible matrices of same orders then $(AB)^{-1}$ is equal to ------ ?

MATHS KHAZANA

SAMPLE PAPER 8

SECTION A

Q1. Let $f:[2,\infty)\to R$ be the function defined by $f(x)=x^2-4x+5,$ then the range of f is

(a) R (b) $[1,\infty)$ (c) $[4,\infty)$ (d) $[5,\infty)$

Q2. Two matrices A and B (both matrices are of same orders $m\times n$) in such a way that total elements in $3A-5B$ is six and number of elements in AB^T is nine : then the order of A.

(a) 1×6 (b) 2×3 (c) 3×2 (d) 6×1

Q3. If A is a square matrix of order 3×3 such that $|A|=5$ then $|adj(2A)|$ is

(a) 25 (b) 625 (c) 1600 (d) 1200

Q4. If $y=\cos^{-1}(\cos 5)$ then y is

(a) 5 (b) $\pi-5$ (c) $2\pi-5$ (d) $\pi+5$

Q5. $f(x)=3\tan x+5$ is discontinuous at

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{11\pi}{6}$ (d) $\frac{3\pi}{2}$

Q6. The maximum value of $4\sin^2 x+3\cos^2 x$ is :

(a) 3 (b) 4 (c) 5 (d) 7

Q7. If $x=t^2, y=t^3$, then $\frac{d^2y}{dx^2}$ is :

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3}{4}$

Q8. The corner points of the feasible region determined by the system of linear inequalities are $(0,0),(4,0),(2,4)$ and $(0,5)$. If the maximum value of $z=ax+by,$ where $a,b>0$ occurs at both $(2,4)$ and $(4,0),$ then

(a) $a=2b$ (b) $2a=b$ (c) $a=b$ (d) $3a=b$

Q9. If is given that the events A and B are such that $P(A)=\frac{1}{4}, P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$. Then $P(B)$ is

MATHS KHAZANA

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{2}{3}$

Q10. The range of $f(x)=\sin^{-1}x+\operatorname{cosec}^{-1}x$ is

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ (c) $[-\pi,\pi]$ (d) $\{-\pi,\pi\}$

Q11. The value of $(\hat{i}\times\hat{j})\cdot\hat{k}+\hat{i}\cdot\hat{j}$ is

(a) -1 (b) 0 (c) 1 (d) 2

Q12. Describe the nature of the family of curves represented by the solution of the differential equation $\frac{dy}{dx}=m$.

(a) Parabola (b) Straight line (c) Circle (d) Ellipse

Q13. Value of : $\int e^x\sqrt{e^x+5}\ dx$

(a) $\frac{3}{2}(e^x+5)^{3/2}+c$ (b) $\frac{3}{2}(e^x+5)^{2/3}+c$ (c) $\frac{2}{3}\sqrt{e^x+5}+c$ (d) $\frac{2}{3}(e^x+5)^{3/2}+c$

Q14. The equation of a line is represented by $5x-3=15y+7=3-10z$ direction cosines of the line are

(a) $5:15:-10$ (b) $6:10:-3$ (c) $6:2:-3$ (d) $-3:4:5$

Q15. Solve : $\int_{-\pi/2}^{\pi/2}\sin^7 x\,dx$

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

Q16. The function $f(x)=\tan^{-1}x-x$

(a) always increasing (b) always decreasing
(c) never increasing (d) neither increasing nor decreasing.

Q17. If $|\vec{a}|=3$ and $-1\le k\le 2$, then $|k\vec{a}|$ lies in the interval :

(a) [0, 6] (b) $[-3,6]$ (c) [3, 6] (d) None of these

Q18. Let $n(\mathrm{A})=4$ $n(\mathrm{B})=p$ and the number of functions from A into B are 81, then p will be

(a) 2 (b) 3 (c) 4 (d) 9

MATHS KHAZANA

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A) :** Value of $\sec^2\left(\tan^{-1} 5\right) + \operatorname{cosec}^2\left(\cot^{-1} 3\right) = 25$

Reason (R) : $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$

Q20. **Assertion (A) :** $f(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous at $x = 0$.

Reason (R) : Function *f* is continuous at $x = 0$ if $\lim_{x \to 0} f(x) = f(0)$

SECTION B

Q21. Find X and Y if 2X + 3Y = $\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and 3X + 2Y = $\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Q22. If $y = e^{e^x} + e^{x^e} + e^{x^x}$ find $\dfrac{dy}{dx}$.

Q23. Evaluate : $\displaystyle\int \frac{\log(\log x)}{x}\, dx$

Q24. Find the particular solution of $e^{dy/dx} = x + 1$, given that $y = 3$ when $x = 0$.

Q25. Find $|\vec{a}|$ and $|\vec{b}|$, **if** $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

SECTION C

Q26. Differentiate w.r.t. x : $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$

Q27. A bag contains 6 red and 8 black balls and another bag contains 8 red and 6 black balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag. Find the probability that the ball drawn is red in colour.

MATHS KHAZANA

Q28. Evaluate : $\int_{\frac{1}{2}}^{\frac{3}{2}} |x \sin \pi x| \, dx$ **OR** Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

Q29. Solve the differential equation : $\left[x \sin^2\left(\frac{y}{x}\right) - y\right] dx + x \; dy = 0$; $y = \frac{\pi}{4}$ when $x = 1$

OR

Find the equation of a curve passing through the point (0,1).If the slope of the tangent to the curve at any point (*x*,*y*) is equal to the sum of the *x* coordinate (abscissa) and the product of the *x* coordinate and *y* coordinate (ordinate)of that point.

Q30. Show that the minimum of Z occurs at more than two points .

Minimise and Maximise $Z = x + 2y$

Subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200;\ x, y \geq 0$.

Q31. If $y = e^{\tan x}$ prove that $(\cos^2 x)\frac{d^2y}{dx^2} - (1 + \sin 2x)\frac{dy}{dx} = 0$

OR

If the function $f(x)$ given by $f(x) = \begin{cases} 3ax + b, & if\ x > 1 \\ 11, & if\ x = 1 \\ 5ax - 2b, & if\ x < 1 \end{cases}$ is continuous at $x = 1$, find the value of *a* and *b*.

SECTION D

Q32. Find the area bounded by the curve $y^2 = 4a^2(x-1)$ and the lines $x = 1$ and $y = 4a$

Q33. Prove that the Greatest Integer Function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = [x]$ is neither one-one nor onto, where $[x]$ denotes the greatest integer less than *or* equal to *x*.

Q34. Show that the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda\ (3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ intersect. Find their point of intersection.

Q35. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations :

$x - y = 3,\qquad 2x + 3y + 4z = 17,\qquad y + 2z = 7$

MATHS KHAZANA

SECTION E

Q36. Read the following passage and then answer the questions given below.

The Indian Cost Guard (ICG) while patrolling, saw a suspicious boat with some men. They were not looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. One of the officer observed that the boat is moving along a plane surface. At an instant, the coordinates of the position of coast guard helicopter and boat are at the points A(2, 3, 5) and B(1, 4, 2) respectively.

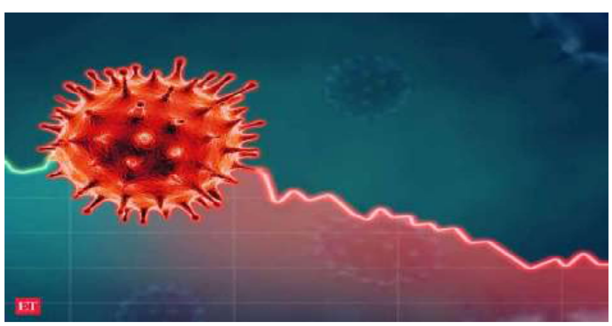
(i) Write the direction cosines of line AB.

(ii) When the position of coast guard helicopter is at the point C(1, 0, –3), then the position of the boat is at the point D(3, –2, 3). Check if the line CD is parallel to line AB. Justify.

Q37. The windows of a newly constructed building are in the form of a rectangle surmounted by an equilateral triangle. The perimeter of each window is 30 *m* as shown.
On the basis of above information answer the following questions :

(i) Explain the area A of rectangular region in terms of x ?
(ii) What is the maximum area A ?
(iii) What are the values of x and y for maximum area A?

Q38. By examining the Covid-19 test report of some laboratory, the probability that a person is diagnosed Covid-19 positive when he is actually suffering from it is 0.99. The probability that the report incorrectly diagnosed a person to be Covid-19 positive on the basis of report is 0.001. In a certain city 300 of 1000 persons suffer from Covid 19.



On the basis of above information answer the following questions :

(i) What is the conditional probability that a person is diagnosed Covid - 19 positive given that he actually has Covid -19 ?

MATHSKHA

(ii) A person is selected at random and is diagnosed with Covid -19. What is the chance that he actually has Covid-19 ?

(iii) MCD wants to keep a check, so an officer during checking selects the report randomly. According to the report, person is diagnosed Covid-19 positive. What is the probability that the person doesn't have actually Covid-19 ?

(iv) What is the total probability of a person being diagnosed Covid-19 positive ?

OR

Let T be the event of person being diagnosed Covid-19 positive and let E_1 and E_2 be the events that he actually has Covid-19 and the actually doesn't have Covid-19 respectively then what is the value of $\sum_{i=1}^{2} P(E_i/T)$?

MATHS KHAZANA

SAMPLE PAPER 9

SECTION - A

Q1. Let $f : A \rightarrow B$ be a bijective function such that range of f is $\{1,2\}$, then $n(A)$ is

(a) 1 (b) 2 (c) 4 (d) 8

Q2. If matrix A has 4 elements and B has p elements in such a way that A + B and AB both are defined, then the order of B be

(a) 2×3 (b) 3×2 (c) 3×3 (d) 2×2

Q3. If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ and $A(adj\ A) = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of k is :

(a) $\sin x \cos x$ (b) 1 (c) 2 (d) 3

Q4. $\tan\left(\cos^{-1} x\right)$ is equal to

(a) $\dfrac{\sqrt{1+x^2}}{x}$ (b) $\dfrac{1}{\sqrt{1-x^2}}$ (c) $\dfrac{\sqrt{1-x^2}}{x}$ (d) $\dfrac{1}{1+x^2}$

Q5. The value of $\dfrac{d(\sin x)}{d(\cos 2x)}$ is equal to :

(a) $-\dfrac{1}{4}\operatorname{cosec} x$ (b) $\dfrac{1}{4\sin x}$ (c) $-4\sin x$ (d) None of these

Q6. The value of 'a' for which $f(x) = \log_a x$ is decreasing on $(0,\infty)$.

(a) $-1 < a < 0$ (b) $0 < a < 1$ (c) $1 < a < \infty$ (d) None

Q7. If $y = \log_{\sqrt{\sin x}}(\sin x)$ then the value of $\dfrac{dy}{dx}$ is :

(a) 0 (b) 1 (c) $\cos x$ (d) $\tan x$

Q8. Feasible region is the set of points which satisfy

(a) the objective functions (b) some of the given constraints
(c) all of the given constraints (d) None of these

Q9. A bag contains 5 white and 4 red balls, 2 balls are drawn from the bag, the probability that both are white is

(a) $\dfrac{4}{9}$ (b) $\dfrac{5}{9}$ (c) $\dfrac{5}{18}$ (d) $\dfrac{2}{9}$

MATHS KHAZANA

Q10. The range of $f(x)=\frac{1}{3}\sin^{-1}x$

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{3},\frac{\pi}{3}\right]$ (c) $\left[-\frac{\pi}{6},\frac{\pi}{6}\right]$ (d) None

Q11. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $|\vec{a}\times\vec{b}|=13$ and $|\vec{a}|=5,$ then $|\vec{b}|$ is

(a) $\frac{5}{13}$ (b) $\frac{13}{5}$ (c) 5 (d) 13

Q12. Solve : $\frac{dy}{dx}=c,$ given that $y(1)=-2$.

(a) $y=c(x-1)-2$ (b) $y=c(x-1)+2$

(c) $y=c(x-1)+3$ (d) $y=-c(x-1)+3$

Q13. Value of : $\int x^3\sin(x^4+1)\,dx$

(a) $-\frac{1}{4}\cos(x^4+1)+C$ (b) $\frac{1}{4}\cos(x^4+1)+c$

(c) $-\cos(x^4+1)+c$ (d) $\cos(x^4+1)+c$

Q14. If α,β,γ are the angles that a line makes with x, y and z axes respectively, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is

(a) 0 (b) 1 (c) 2 (d) 3

Q15. Value of : $\int_{-\pi/2}^{\pi/2}|\sin x|\,dx$

(a) 0 (b) 1 (c) 2 (d) 4

Q16. The value of 'a' for which $f(x)=a^x$ is increasing on R is

(a) $0<a<1$ (b) $\frac{1}{2}<a<\frac{5}{2}$ (c) $1<a<\infty$ (d) None

Q17. If $\vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$ then :

(a) $\vec{a}\parallel(\vec{b}\times\vec{c})$ (b) $\vec{a}\perp(\vec{b}\times\vec{c})$ (c) $\vec{b}\perp(\vec{a}\times\vec{c})$ (d) $\vec{b}\parallel(\vec{a}\times\vec{c})$

Q18. Let $A=\{1,2,3\}$ $B=\{4,5\}$. The number of injection function from A into B are

(a) 6 (b) 3 (c) 2 (d) 0

MATHS KHAZANA

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q19. **Assertion (A)** : $\sin^{-1}\frac{5}{3}+\cos^{-1}\frac{5}{3}=\frac{\pi}{2}$

Reason (R) : $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2} \quad \forall \quad x\in R$

Q20. **Assertion (A) :** $f(x)=|x-5|$ is not differentiable at $x=5$.

Reason (R) : LHD $(\text{At } x=5)\neq$ RHD $(\text{At } x=5)$ for $f(x)=|x-5|$

SECTION - B

Q21. If $f(x) = x^2 - 5x + 7$, find $f(A)$ if $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

Q22. Differentiate the functions with respect to x : $(\sin x)^{\log x}$

Q23. Solve : $\int\frac{8x+13}{\sqrt{4x+7}}\,dx$

Q24. Solve the differential equations : $\log\left(\frac{dy}{dx}\right)=ax+by$

Q25. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2\hat{i}-3\hat{k}$ and $4\hat{j}+2\hat{k}$.

SECTION - C

Q26. Differentiate w.r.t. x : $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$

Q27. If A and B are two independent events such that $P(\overline{A}\cap B)=\frac{2}{15}$ and $P(A\cap\overline{B})=\frac{1}{6}$, then find P(B).

MATHS KHAZANA

Q28. Solve : $\int_0^{\pi/2}(2\log\sin x-\log\sin 2x)\,dx$ **OR** Solve : $\int_{-\pi/2}^{\pi/2}(\sin|x|+\cos|x|)\,dx$

Q29. Solve differential equation : $x^2dy+\left(xy+y^2\right)dx=0$; $y=1$ when $x=1$

OR

Show that the family of curves for which the slope of the tangent at any point (x,y) on it is $\frac{x^2+y^2}{2xy}$, is given by $x^2-y^2=cx$.

Q30. Solve the following Linear Programming problems graphically :
Maximise $Z=3x+2y$ Subject to $x+2y\le 10$, $3x+y\le 15$, $x,y\ge 0$.

Q31. If $y=x^3\log\left(\frac{1}{x}\right)$, then prove that $x\frac{d^2y}{dx^2}-2\frac{dy}{dx}+3x^2=0.$

OR

Find the value of *a* when the function is continuous at $x=0$. $f(x)=\begin{cases}\frac{1-\cos 4x}{x^2} & ,if\ x<0\\ a & ,if\ x=0\\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & ,if\ x>0\end{cases}$

SECTION - D

Q32. Find the area lying above the *x*- axis and under the parabola $y=4x-x^2$

Q33. Let $\mathrm{A}=\{x\in \mathrm{R}:-1\le x\le 1\}$. Show that $f:A\to A$ given by $f(x)=x|x|$ is a one -one and onto function .

Q34. Show that the lines $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$ and $\frac{x+2}{3}=\frac{y-1}{3}=\frac{z+1}{-2}$ do not intersect.

Q35. Express the matrix $\begin{bmatrix}6 & 1 & -5\\ -2 & -5 & 4\\ -3 & 3 & -1\end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

MATHS KHAZANA

SECTION - E

Q36. Read the following passage and the answer the questions given below.
A general election of Lok Sabha is a gigantic exercise. About 958 million people were eligible to vote and voter turnout was about 72%, the highest ever.
Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2024. A relation 'R' is defined on I as follows.

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election - 2024$\}$.

ONE - NATION
ONE - ELECTION

FESTIVAL OF DEMOCRACY

GENERAL ELECTION - 2024

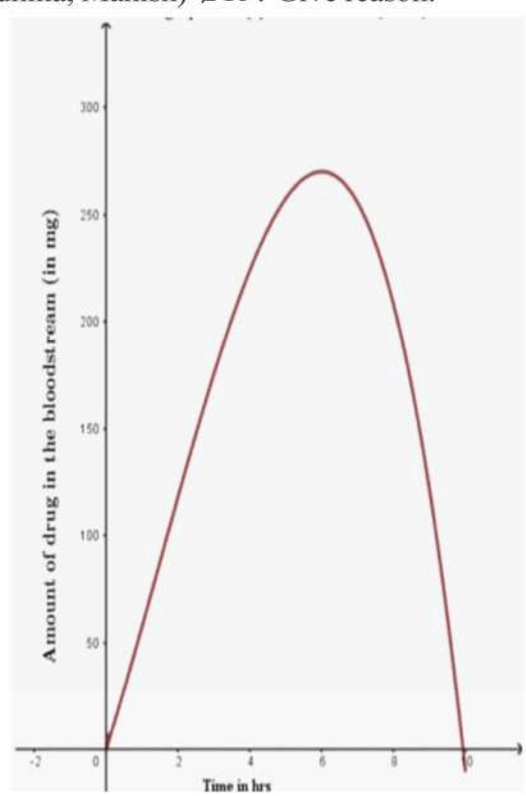
(i) Check if R is reflexive or symmetric or transitive. Give reasons to support your answer.

(ii) Mr. Ravi exercised his voting right in general election - 2024.While his brother (having voting right), Mr. Manish went to have fun at a nearby mall. Can we have (Ravi, Manish) $\in R$? Give reason .
If Miss. Radhika (having voting right) goes with Mr. Manish to the mall skipping the voting exercise, then is it correct to say (Radhika, Manish) $\notin R$? Give reason.

Q37. Answer the questions based on the given information .
A medicinal drug administered into a human body requires some time to produce its effect on the body . The amount (in mg) of a certain medicinal drug in the bloodstream at t hours after administering the drug to an individual is given by the function

$C(t) = -t^3 + 4.5t^2 + 54t, \quad 0 \le t \le 10$

Shown below is the graph of C (t) in the interval [0, 10] .



(i) Find the rate at which the amount of drug is changing in the bloodstream at 5 hours after administering the drug . Show your work .

(ii) Show that the function C (t) is strictly increasing in the interval (3, 4) .

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Q38. A shopkeeper sells three types of flower seeds A_1, A_2, A_3. They are sold as mixture where the proportions are 3 : 2 : 1 respectively . The germination rates of the three types of seeds are 45% , 60% and 35 % .

A seed is randomly chosen by a customer .

Based on the above information answer the following :

(i) Calculate the probability that it will germinate , given that the seed is of type A_2 .

(ii) Calculate the probability that it will not germinate given that the seed is of type A_3 .

(iii) (a) Calculate the probability of the randomly chosen seed to germinate .

OR

(b) Calculate the probability that it is of type A_2 given that a randomly chosen seed germinate .

MATHS KHAZANA

SAMPLE PAPER 10

SECTION - A

Q1. Let $A = \{x, y\}$, then the number of reflexive and symmetric relations on A are

(a) 1 (b) 2 (c) 4 (d) 8

Q2. If A be a matrix of order $m \times n$ such that $2A - A^T = I$ and AA^T has 9 elements then the orders of A

(a) 9×1 (b) 1×9 (c) 2×2 (d) 3×3

Q3. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and B is the additive inverse of A then $|AB|$.

(a) -81 (b) 81 (c) -729 (d) 729

Q4. Value of $\cos\left(\tan^{-1}\frac{3}{7}\right)$ is

(a) $\frac{4}{7}$ (b) $\frac{3}{7}$ (c) $\frac{3}{\sqrt{58}}$ (d) $\frac{7}{58}$

Q5. If $f(x) = |x + a|$ is not differentiable at $x = 5$ then the value of a is

(a) 0 (b) 5 (c) -5 (d) None

Q6. The maximum value of $f(x) = -|x + 2|$ is :

(a) 2 (b) -2 (c) 0 (d) None

Q7. If $y = a\sin x + b\cos x,$ then value of $y^2 + \left(\frac{dy}{dx}\right)^2$ is :

(a) $a^2 + b^2$ (b) ab (c) $a^2 - b^2$ (d) $\frac{1}{a^2 + b^2}$

Q8. The corner points of the feasible region determined by the system of linear inequalities are $(0,0),(5,0),(6,5),(6,8),(4,10)$ and $(0,8)$ Let $z = 3x - 4y$ be the objective function. Mini mum of z occurs at

(a) $(0,0)$ (b) $(0,8)$ (c) $(5,0)$ (d) $(4,10)$

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Q9. If P($\overline{A}$) = 0.7, P($\overline{B}$) = 0.7 and P(B/A) = 0.5, then P(A/B) is

(a) 0.30 (b) 0.40 (c) 0.45 (d) 0.50

Q10. $-\frac{2\pi}{5}$ is the principal value of

(a) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$ (b) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$

(c) $\sec^{-1}\left(\sec\frac{7\pi}{5}\right)$ (d) $\cot^{-1}\left(\cot\frac{7\pi}{5}\right)$

Q11. The value of 'p' which makes $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ parallel is

(a) -3 (b) -1 (c) $-\frac{1}{3}$ (d) -9

Q12. Solution of : $\frac{dy}{dx}=e^{x+y}$

(a) $e^{x}+e^{-y}+c=0$ (b) $e^{-x}+e^{-y}+c=0$

(c) $e^{-x}+e^{y}+c=0$ (d) $e^{-x}-e^{-y}+c=0$

Q13 . Value of : $\int_0^1 xe^{x^2}\,dx$

(a) $-\frac{1}{2}(e-1)$ (b) $-\frac{1}{2}(e+1)$ (c) $\frac{1}{2}(e-1)$ (d) $\frac{1}{2}(e+1)$

Q14. The direction ratio of the line $x=5,\ \frac{y-3}{4}=\frac{2-z}{1}$ is :

(a) $0,4,-1$ (b) $0,-4,-1$ (c) $1,4,-1$ (d) $1,4,1$

Q15. Solve : $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$

(a) $2\log 3$ (b) $\frac{1}{2}\log 3$ (c) $2-\log 3$ (d) 0

Q16. The value of a for which the function $f(x)=x^2-2ax+6, x>0$ is strictly increasing :

(a) $o<a<1$ (b) $a\le 0$ (c) $a>1$ (d) None of these

Q17. Let $\vec{a}=-2\hat{i}+\hat{j}, \vec{b}=\hat{i}+2\hat{j}$ and $\vec{c}=4\hat{i}+3\hat{j}$, the values of x and y such that $\vec{c}=x\vec{a}+y\vec{b}$ is :

MATHS KHAZANA

(a) $x = -1, y = 2$ (b) $x = 1, y = -2$

(c) $x = -1, y = -2$ (d) $x = 1, y = 2$

Q18. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ The number of injective functions from A into B are

(a) 3 (b) 6 (c) 12 (d) 24

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **<u>Assertion (A)</u>** **:** Domain of $\sin^{-1}(3x+4)$ is $\left[-\frac{5}{3}, -1\right]$

<u>Reason (R)</u> **:** $\sin^{-1} x$ is defined $\forall\, x \in [-1, 1]$

Q20. **<u>Assertion (A)</u>** **:** If $y = \log\left(x + \sqrt{1+x^2}\right)$ then $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

<u>Reason (R)</u> **:** $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

SECTION - B

Q21. If A and B symmetric matrices, prove that AB − BA is a skew symmetric matrix.

Q22. Differentiate with respect to x : $\cos^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}, \; -\frac{\pi}{4} < x < \frac{\pi}{4}$

Q23. Evaluate : $\int \frac{\sec^4 x}{\sqrt{\tan x}}\, dx$

Q24. Solve the differential equations : $\frac{dy}{dx} + 3y = e^{-2x}$

Q25. Show that the area of a parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} - 4\hat{k}$ is $5\sqrt{3}$ **.**

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SECTION - C

Q26. If $xy\log(x+y)=1$, prove that $\frac{dy}{dx}=-\frac{y(x^2y+x+y)}{x(xy^2+x+y)}$

Q27. Let A and B be two independent events such that the probability is $\frac{1}{8}$ that they will occur simultaneously and $\frac{3}{8}$ that neither of them will occur . Find P (A) and P(B).

Q28. Solve : $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$ **OR** Solve : $\int_0^{2\pi}\frac{1}{1+e^{\sin x}}\,dx.$

Q29. Solve the differential equation : $\left(xe^{y/x}+y\right)\,dx = x\,\,dy$, $y\,(1)=1$

OR

Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point.

Q30. Show that the minimum of Z occurs at more than two points .
Minimise and Maximise $Z=5x+10y$ Subject to $x+2y\le 120,\ x+y\ge 60,\ x-2y\ge 0,\ x,y\ge 0$

Q31. Prove that : $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]=\sqrt{a^2-x^2}$

OR

Show that $f(x)=\begin{cases}\frac{\sin 5x}{\tan 3x}, & \text{if } x<0\\ \frac{5}{3}, & \text{if } x=0\\ \frac{\log(1+5x)}{e^{3x}-1}, & \text{if } x>0\end{cases}$ is continuous at $x = 0$.

SECTION - D

Q32. Prove that the product of matrices $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$ and $\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$ is the null matrix, when θ and ϕ differ by an odd multiple of $\pi/2$.

Q33. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A$(2,0)$, B $(4,5)$ and C $(6,3)$.

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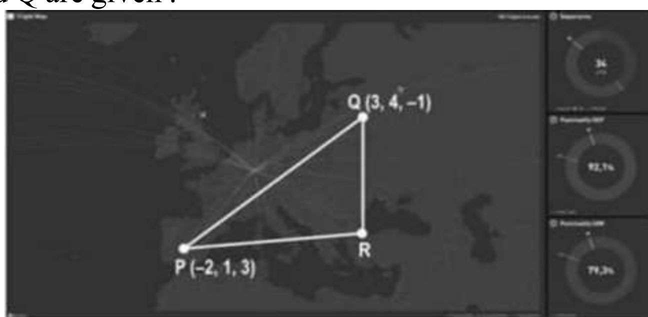
Q34. Let $f : N \to N$ be defined by $f(n) = \begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$, show that f is a bijective.

Q35. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

SECTION - E

Q36. Answer the questions based on the given information .
The flight path of two airplanes in a flight simulator game are shown below . The coordinates of the airports P and Q are given .



Airplane 1 flies directly from P to Q .
Airplane 2 has a layover at R and then flies to Q .

The path of Airplane 2 from P to R can be represented by the vector $5\hat{i} + \hat{j} - 2\hat{k}$.

(i) Find the vector that represents the flight path of Airplane 1 . Show your steps .

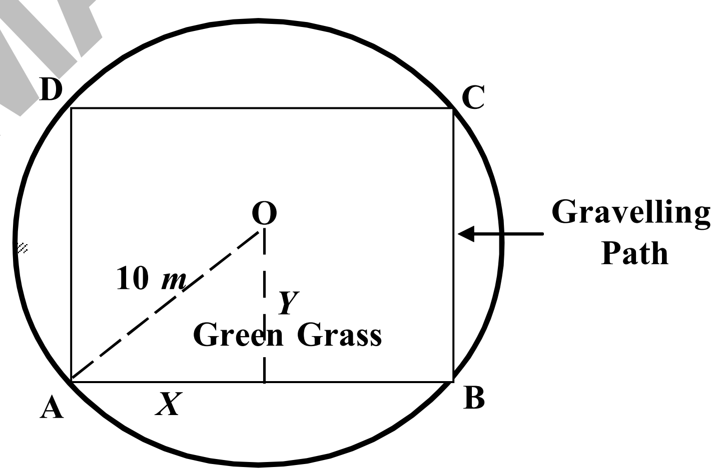
(ii) Write the vector representing the path of Airplane 2 from R to Q. Show your steps .

(iii) (a) What is the angle between the flight paths of Airplane 1 and Airplane 2 just after takeoff ? Show your work .

OR

(b) Consider that Airplane 1 started the flight with a full fuel tank . Find the position vector of the point where one third of the fuel runs out if the entire fuel is required for the flight . Show your work .

Q37. An architect design a garden in society. The garden is in the shape of rectangle inscribed in a circle of radius 10 *m* as shown in given figure .



On the basis of above information answer the following questions :

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(i) $2x$ and $2y$ represents the length and breadth of the rectangular part, then establish the relation between the variables.

(ii) What is the area of the green grass A expressed as a function of x ?

(iii) What is the maximum value of area A ?

Q38. One day Mohan went to village to attend his cousin brother's marriage party. He went along with his father, mother and sister. He wants one family photograph. So, he requested to photog rapher for the same. All are line up at random for a family photograph.

On the basis of above information answer the following questions :

(i) Find the probability that son is at one end, given that father and mother are in the middle.

(ii) Find the probability that mother is at left end, given that son and daughter are together.

(iii) Find the probability that father and mother are in the middle, given that daughter is at right end.

(iv) Find the probability that mother and son are standing together, given that father and daughter are standing together.

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SAMPLE PAPER 11

SECTION - A

Q1. Given set $A=\{1,2,3\}$ and a relation $R=\{(2,2),(1,2),(2,1)\}$, the relation R will be

(a) Reflexive if $(1,1)$ is added (b) Symmetric if $(2,3)$ is added

(c) Transitive if $(1,1)$ is added (d) Symmetric if $(3,2)$ is added

Q2. If $A=\begin{bmatrix}0 & 1 & 3\\ 5 & 1 & 2\end{bmatrix}$ $B=\begin{bmatrix}1 & 2\\ 3 & 1\\ 4 & 0\end{bmatrix}$ and $BA=(b_{ij})$ then $b_{21}+b_{31}$.

(a) 4 (b) 5 (c) 6 (d) 8

Q3. If $A=\begin{bmatrix}2 & \lambda & -3\\ 0 & 2 & 5\\ 1 & 1 & 3\end{bmatrix}$, then A^{-1} does not exists for

(a) $\lambda=2$ (b) $\lambda=-\frac{8}{5}$ (c) $\lambda=-8$ (d) None of these

Q4. The domain of function $f(x)=\sin^{-1}(1-x^2)$

(a) $[0,\sqrt{2}]$ (b) $[-\sqrt{2},0]$ (c) $[-\sqrt{2},\sqrt{2}]$ (d) None

Q5. The value of k which makes the function $f(x)=\begin{cases}x\sin\frac{1}{x}, & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$ continuous at $x=0$ is :

(a) 0 (b) 1 (c) -1 (d) None of these

Q6. The function $f(x)=|\cos x|$ is :

(a) Everywhere continuous and differentiable

(b) Everywhere continuous but not differentiable at $(2n+1)\frac{\pi}{2}, n\in Z$

(c) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n\in Z$

(d) None of these

Q7. If $\int \sec^{2n+1}x\tan x\,dx=\frac{\sec^5 x}{5}+c$ then the value of n is

(a) 1 (b) 2 (c) 3 (d) 4

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Q8. If $\overrightarrow{AB}\times\overrightarrow{AC}=2\hat{i}-4\hat{j}+4\hat{k}$, then the area of ΔABC is

(a) 3 *Sq. unit* (b) 4 *Sq. unit*
(c) 16 *Sq. unit* (d) 9 *Sq. unit*

Q9. Value of : $\sin\left(\frac{dy}{dx}\right)=1$

(a) $y=\frac{\pi x}{2}+c$ (b) $y=\frac{2}{\pi}x+c$ (c) $y=\pi x+c$ (d) $y=\frac{x}{\pi}+c$

Q10. If $y=\log_x 5$ then $\frac{dy}{dx}$ is

(a) $\frac{-1}{\log_x 5}$ (b) $\frac{-\log 5}{\log_e x}$ (c) $\frac{-\log 5}{(\log_e x)^2}$ (d) $\frac{-\log 5}{x(\log_e x)^2}$

Q11. The value of $\int_{-\pi/2}^{\pi/2}\frac{dx}{\sin^3 x+\sin x}$ is

(a) 1 (b) 0 (c) $\frac{\pi^2-3}{2}$ (d) π

Q12. The line $x=1+5\mu,\ y=-5+\mu,\ z=-6-3\mu$ passes through which of the following points ?

(a) $(1,-5,6)$ (b) $(1,5,6)$ (c) $(1,-5,-6)$ (d) $(-1,-5,6)$

Q13. If $|\vec{a}|=3, |\vec{b}|=4$ and $\vec{a}\perp\vec{b}$, then $|\vec{a}-\vec{b}|$ is equal to :

(a) 6 (b) 5 (c) 4 (d) 3

Q14. If f is a real valued function defined as $f(x)=\frac{e^{2x}-1}{e^{2x}+1}$ is :

(a) always increasing (b) always decreasing
(c) both increasing and decreasing (d) none of these

Q15. Let $A=\{1,2,3\}, B=\{4,5\}$ which subset of $A\times B$ is not a function from A to B.

(a) $\{(1,4),(2,5),(3,4)\}$ (b) $\{(1,4),(2,5),(3,5)\}$
(c) $\{(1,4),(2,4),(3,5),(2,5)\}$ (d) $\{(1,5),(2,5),(3,4)\}$

Q16. If $\cos^{-1}\frac{x}{13}+\sec^{-1}\left(\frac{13}{5}\right)=\frac{\pi}{2}$, then the value of x is

(a) 10 (b) 11 (c) 12 (d) 13

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Q17. If A and B are two events such that $P(A) \neq 0$ and $P(B) \neq 1$, then $P\left(\frac{\bar{A}}{B}\right) =$

(a) $1 - P\left(\frac{A}{B}\right)$ (b) $1 - P\left(\frac{\bar{A}}{B}\right)$ (c) $\frac{1 - P(A \cup B)}{P(B)}$ (d) $\frac{P(\bar{A})}{P(B)}$

Q18. Which of the following statements is false ?
- (a) The maximum (or minimum) solution of the objective function occurs at the vertex of t he feasible region
- (b) The feasible region is always a concave region
- (c) If two corner points produce the same maximum (or minimum) value of the objective function, then every point on the line segement joining these points will also give the same maximum (or minimum) value.
- (d) None of these

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A) :** All trigonometric functions have their inverses over their respective domains .

Reason (R) : The inverse of $\tan^{-1} x$ exists for some $x \in R$.

Q20. **Assertion (A) :** The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular , when $\vec{b}_1 . \vec{b}_2 = 0$.

Reason (R) : The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by

$$\cos\theta = \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

SECTION - B

Q21. Construct a 3×3 matrix, whose elements a_{ij} are given by in each of the following :

$$a_{ij} = \frac{1}{2}|-3i + j|$$

Q22. Differentiate w.r.t. x : $\cos^{-1}(1 - 2x^2)$

Q23. Evaluate : $\int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx$

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Q24. Solve $: dy = e^{2x+y}\, dx$, $y(0) = 0$.

Q25. Find a unit vector in the direction of the resultant of the vectors $\hat{i} - \hat{j} + 3\hat{k},\ 2\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} + 2\hat{j} - 2\hat{k}$

SECTION - C

Q26. Differentiate $\tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$ w.r.t. $\tan^{-1}\left(\dfrac{2x}{1 - x^2}\right)$

Q27. A bag contains 3 red and 4 white balls. Find the probability distribution of the number of red balls drawn when 3 balls are drawn with replacement.

Q28. Evaluate : $\displaystyle\int_{-4}^{4} x^3 \cos\left(\frac{e^x + 1}{e^x - 1}\right) dx$ **OR** Evaluate : $\displaystyle\int_{0}^{3/2} |x \cos \pi x|\, dx$

Q29. Maximise $Z = -x + 2y$, subject to the constraints : $x \ge 3, x + y \ge 5, x + 2y \ge 6,\ y \ge 0$

Q30. Solve : $2xy + y^2 - 2x^2 \dfrac{dy}{dx} = 0$; $y = 2$ when $x = 1$

Q31. If $y = \left(\sec^{-1} x\right)^2, x > 0$ show that $x^2\left(x^2 - 1\right)\dfrac{d^2y}{dx^2} + \left(2x^3 - x\right)\dfrac{dy}{dx} - 2 = 0$

SECTION - D

Q32. Using integration, find the area of triangle ABC, whose vertices are A (2, 5), B (4, 7) and C (6, 2)

Q33. Given a non - empty set X, consider P (X) which is the set of all subsets of X . Define R in P (X) as follows: For subsets $A, B \in P(X)$: we have $A\ R\ B \Leftrightarrow A \subseteq B$ is R an equivalence relation on P (X)? Justify your answer .

Q34. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane.

Q35. Verify $A(adj\ A) = (adj\ A)A = |\ A\ |\ I$: where $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$.

SECTION - E

Q36. Read the following passage and answer the questions given below. Utkarsh was doing a survey on a school. Theme of the survey was 'the average number of hours spent on study' by students selected at random. At the end of survey, he prepared the following report related to the data. Let X denotes the average number of hours spent on study by students. The probability that X can take the values x, has the following form, where k is some constant.

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$$P(X=x)=\begin{cases}0.2, \text{ if } x=0\\ kx, \text{ if } x=1 \text{ or } 2\\ k(6-x), \text{ if } x=3 \text{ or } 4\\ 0, \text{ otherwise}\end{cases}$$

(i) What is the value of k?

(ii) What is the probability that the average study time of students is not more than 1 hour?

(iii) What is the probability that the average study time to students is at least 3 hours?

OR

(iii) What is the probability that the average study time of students is at least 1 hour?

Q37. Consider the function $f: A \to B$ defined by $f(x)=x^2$

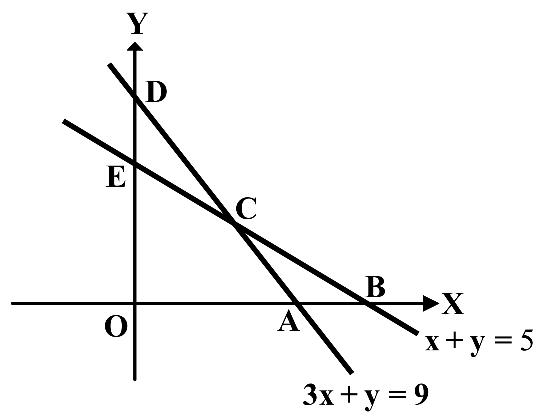
(i) If A be the set of real numbers what should be B to make $f: A \to B$ surjective.

(ii) If $A=\{1,2,5\}$ then how many elements should be in set B to make f bijective

(iii) If $A=\{1,2,3\}$ then how many minimum elements are required in set B to make f one-one and into.

(iv) If $A=\{-2,-1,1,2,3\}$, $B=\{1,2,3,.....,10\}$ and $f: A \to B$ $f(x)=x^2$. Check bijectivity ?

Q38. In LPP if constraints are $3x+y \le 9,\ x+y \le 5,$ $x \ge 0\ \ y \ge 0$ and objective function $Z = x+2y$ then answer the followings :



(i) Which point doesn't lie in the feasible region

(a) $(1,1)$ (b) $(2,1)$ (c) $(2,3)$ (d) $(5,2)$

(ii) Feasible region in the graph

(a) Δ CDE (b) Δ ABC

(c) Quadrilateral OAEC (d) unbounded region

(iii) Intersection point of lines $3x+y=9$ and $x+y=5$ is

(a) $(3,2)$ (b) $(1,4)$ (c) $(0,5)$ (d) $(2,3)$

(iv) Maximum value of objective function

(a) 3 (b) 14 (c) 8 (d) 10

(v) If objective function is $Z = 2x + 3y$ then the maximum value of Z

(a) 5 (b) 10 (c) 15 (d) 20

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SAMPLE PAPER 12

SECTION - A

Q1. Let $A = \{1, 2, 3\}$, Then, the number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is

(a) 1 (b) 2 (c) 3 (d) 4

Q2. If $A = BX$ and $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ then X is

(a) $\begin{pmatrix} 2 & 4 \\ 3 & -5 \end{pmatrix}$ (b) $\frac{1}{2}\begin{pmatrix} -2 & 4 \\ 3 & 5 \end{pmatrix}$ (c) $\frac{1}{2}\begin{pmatrix} 2 & 4 \\ 3 & -5 \end{pmatrix}$ (d) None

Q3. The product of a matrix and its transpose is an identity matrix. The determinant value of this matrix is

(a) 0 (b) ± 2 (c) ± 1 (d) ± 3

Q4. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $xy + yz + zx$ is

(a) 1 (b) 3 (c) 2 (d) 5

Q5. The function $f(x) = |x - 3|$ is continuous on

(a) $R - \{3\}$ (b) R (c) $[0, \infty)$ (d) $(-\infty, 0]$

Q6. The minimum value of curve $y = xe^x$ is :

(a) $-1/e$ (b) $1/e$ (c) $-e$ (d) e

Q7. Solve : $\int_0^{\pi} \frac{1}{1 + e^{\cos x}} dx$

(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) 0

Q8. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

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Q9. The equation of the curve passing through $(3, 9)$ and satisfying the differential equation $\frac{dy}{dx} = x + \frac{1}{x^2}$ is :

(a) $6xy = 3x^3 - 29x + 6$ (b) $6xy = 3x^3 + 29x - 6$

(c) $6xy = 3x^3 - 29x - 6$ (d) $6xy = 3x^3 + 29x + 6$

Q10. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to :

(a) $\frac{4x^3}{1+x^4}$ (b) $-\frac{4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $-\frac{4x^3}{1-x^4}$

Q11. The equation of line passing through the point $(3,4,5)$ and parallel to z-axis is

(a) $\frac{x-3}{1} = \frac{y-4}{0} = \frac{z-5}{1} = \lambda$ (b) $\frac{x-3}{0} = \frac{y-4}{0} = \frac{z-5}{1} = \lambda$

(c) $\frac{x-3}{1} = \frac{y-4}{1} = \frac{z-5}{1} = \lambda$ (d) $\frac{x+3}{0} = \frac{y+4}{0} = \frac{z+5}{1} = \lambda$

Q12. If $\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{a^2-b^2}} \cos^{-1}\left(\frac{b+a\cos x}{a+b\cos x}\right)$ then the value of $\int_0^{\pi} \frac{dx}{a+b\cos x}$ is :

(a) $\frac{\pi}{2\sqrt{b^2-a^2}}$ (b) $-\frac{1}{\sqrt{a^2-b^2}}$

(c) $\frac{\pi}{\sqrt{a^2-b^2}}$ (d) $\frac{2ab}{\sqrt{a^2-b^2}}$

Q13. If $\vec{a} = 6\hat{i} + 3\hat{k} - 2\hat{j}$ then the vector component of $\vec{a}$ in the direction of y-axis is:

(a) $2\hat{j}$ (b) -2 (c) 2 (d) $-2\hat{j}$

Q14. The value of 'a' for which $f(x) = \log_a x$ is decreasing on $(0, \infty)$.

(a) $-1 < a < 0$ (b) $0 < a < 1$

(c) $1 < a < \infty$ (d) None

Q15. If $f : \mathrm{R} \to \mathrm{A}$ given by $f(x) = x^2 - 6x + 12$ is surjective function, then the set A is

(a) $(3, \infty)$ (b) $(-\infty, 3)$ (c) $[3, \infty)$ (d) $(-\infty, 3]$

Q16. The simplified expression of $\sin(\cot^{-1} x)$.

(a) $\frac{x}{\sqrt{1+x^2}}$ (b) $\sqrt{1+x^2}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{\sqrt{1+x^2}}{x}$

MATHS KHAZANA

Q17. If A and B be two events such that $P(A)=0.6, P(B)=0.2$ and $P(A/B)=0.5$, then $P(A'/B')$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{8}$ (d) $\frac{6}{7}$

Q18. The feasible region, for the inequalities $x \geq 0, 2x+3y \leq 5$ and, $y \geq 0$, lies in

(a) I Quadrant (b) II Quadrant
(c) III Quadrant (d) IV Quadrant

ASSERTION - REASON BASED QUESTIONS

In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. **Assertion (A) :** Two coins are tossed simultaneously . The probability of getting two heads , if it is known that at least one head comes up , is $\frac{1}{3}$.

Reason (R) : Let E and F be two events with a random experiment , then $P(F/E)=\frac{P(E\cap F)}{P(E)}$

Q20. **Assertion (A) :** $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}\, dx = 3$

Reason (R) : $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

SECTION - B

Q21. If A and B are symmetric matrices of the same order, then show that AB - BA is a skew-symmet ric matrix.

Q22. Check the differentiability of $f(x)=x|x|$ at $x=0$

Q23. Evaluate : $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\, dx$

Q24. Solve: $\sqrt{1+x^2}\,dy+\sqrt{1+y^2}\,dx=0$.

Q25. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=5, |\vec{b}|=12, |\vec{c}|=13,$ then find $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

MATHS KHAZANA

SECTION - C

Q26. Differentiate $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Q27. A random variable X has the following probability distribution **:**

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (*i*) k (*ii*) P(X < 3) (*iii*) P(X > 6) (*iv*) P(0 < X < 3)

Q28. Evaluate $\int_{-\pi/2}^{\pi/2} \log\left[\frac{ax^2-bx+c}{ax^2+bx+c}\right]dx$ **OR** Evaluate **:** $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\, dx$

Q29. Minimise and Maximise $Z = x + 2y$

Subject to $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200;\ x, y \geq 0$.

Q30. Solve : $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\} y\ dx=\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\} x\ dy$

Q31. If $x = \sin t, y = \sin pt,$ then prove that $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$

SECTION - D

Q32. Sketch the region bounded by the lines $2x + y = 8,\ y = 2,\ y = 4$ and the y-axis. Hence, obtain its area using integration.

Q33. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is bijective .

Q34. A line with direction ratios < 2, 1, 2 > meets each of the lines given thye equations $x = y + a = z$ and $x + a = 2y = 2z$ Find the coordinates of each of these points of intersection

Q35. Find the inverse of the matrix and verify that $A^{-1}A = I_3$, where $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

SECTION - E

Q36. A bag contains 6 red and 8 black balls and another bag contains 8 red and 6 black balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag.

(i) Find the probability that the ball drawn is red in colour.

(ii) Find the probability that black ball is transferred from bag 1 to bag 2 if it is given that red ball has been drawn from bag 2

MATHS KHAZANA

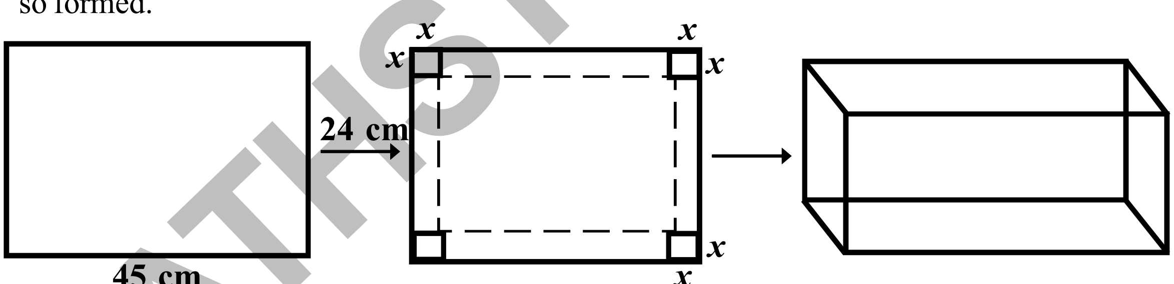
Q37. Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1,2,3,4,5,6\}$. Let A be the set of players while B be the $A=\{S,D\}, B=\{1,2,3,4,5,6\}$.
Using given information answer the followings :

(i) Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible ?

(ii) Raji wants to know the number of functions from A to B. How many number of funtions are possible ?

(iii) Let $R: B \to B$ be defined by $R = \{(x, y) : y \text{ is divisible by } x\}$. Check whether R is reflexive , symmetric or transitive.

Q38. Three friends A, B and C are given a rectangular sheet of sides 45 cm and 24 cm. They are asked to work independently and form an open box by cutting the squares of equal length from all the four corners as shown and folding up the flaps, they want to check the volume of boxes so formed.



On the basis of above information answer the following questions :

(i) If the volume of the box is to be maximised then define volume in terms of x

(ii) What is the value of x for maximum volume ? Also give the value of maximum volume?

MATHS KHAZANA

SAMPLE PAPER 13

SECTION - A

Q1. The range of $f(x)=2-3|x+1|$

(a) $[3,\infty)$ (b) $(-\infty,3]$ (c) $(-\infty,2]$ (d) $[2,\infty)$

Q2. If order of A, B and C are 4×3, 5×4 and 3×7 respectively then, order of $C'(A'\times B')$ is

(a) 7×5 (b) 4×5 (c) 4×3 (d) 5×7

Q3. A and B are two matrices of order 2×2 such that $|A|=5$ $|B|=-3$ then the value of $|adj(AB)|$

(a) -25 (b) -10 (c) -15 (d) -30

Q4. The domain of function $f(x)=\cos^{-1}(3x^2-1)$

(a) $[-\sqrt{2},\sqrt{2}]$ (b) $\left[-\sqrt{\frac{2}{3}},0\right]$ (c) $\left[-\sqrt{\frac{2}{3}},\sqrt{\frac{2}{3}}\right]$ (d) $[-\sqrt{3},\sqrt{3}]$

Q5. The value of 'k' for which the function $f(x)$ is continuous at $x=3$:

$$f(x)=\begin{cases}\dfrac{(x+3)^2-36}{x-3}, & x\neq3\\ 2k+1, & x=3\end{cases}$$

(a) $\frac{11}{2}$ (b) $\frac{9}{2}$ (c) $\frac{13}{2}$ (d) $\frac{15}{2}$

Q6. Maximum slope of the curve $y=-x^3+3x^2+9x-27$ is :

(a) 0 (b) 12 (c) 16 (d) 32

Q7. Value of : $\int_{-2}^{2}\frac{x^3}{1+x^2}\,dx$

(a) $-\frac{4}{9}$ (b) $-\frac{2}{9}$ (c) 0 (d) $\frac{1}{9}$

Q8. If $\hat{i},\hat{j}$ and $\hat{k}$ are three mutually perpendicular vectors, then the value of $\hat{j}.(\hat{k}\times\hat{i})$ is :

(a) -1 (b) 0 (c) 1 (d) None of these

MATHS KHAZANA

Q9. The differential equation $y\frac{dy}{dx}+x=a$, a is any constant, represents :

(a) a set of ellipses
(b) a set of circles having centres on x-axis
(c) a set of circles having centres on y-axis
(d) a set of circles having centres at the origin

Q10. The differential coefficient of $\log_{\sqrt{x}}\sqrt{x}$ is :

(a) 0 (b) 1 (c) -1 (d) $\frac{1}{x}$

Q11. The points $A(1,2,3), B(-1,-2,-3)$ and $C(2,3,2)$ are three vertices of a paralleogram ABCD. The equation of CD is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{2}$
(b) $\frac{x+2}{1}=\frac{y+3}{2}=\frac{z-2}{3}$
(c) $\frac{x}{2}=\frac{y}{3}=\frac{z}{2}$
(d) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-2}{3}$

Q12. Value of : $\int_{-1}^{1}\frac{|x|}{x}dx$

(a) -1 (b) 0 (c) 1 (d) 2

Q13. If $\overrightarrow{AB}=3\hat{i}+2\hat{j}-\hat{k}$ and the coordinate of A are $(4,1,1)$, then the coordinates of B is :

(a) $(7,-3,0)$ (b) $(7,3,0)$ (c) $(-7,3,0)$ (d) $(-7,-3,0)$

Q14. The minimum value of 'k' for which $f(x)=\sin x-kx+7$ is decreasing.

(a) 0 (b) -1 (c) 1 (d) 2

Q15. The domain of function $f(x)=\sqrt{9-x^2}+\sqrt{x-2}$ is

(a) $[-3,3]$ (b) $(-\infty,2]$ (c) $[2,\infty)$ (d) $[2,3]$

Q16. If $\operatorname{cosec}^{-1}\frac{5}{4}+\operatorname{cosec}^{-1}\frac{5}{x}=\frac{\pi}{3}+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ then the value of x is

(a) 2 (b) 3 (c) 4 (d) 5

Q17. Two cards are drawn from a well shuffled pack of 52 cards one after the other with replacement, the probability that one of these is a queen and the other a king of opposite shade is

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{1}{169}$ (d) $\frac{2}{169}$

MATHS KHAZANA

Q18. In an LPP, the objective function is always _____

(a) Non-linear (b) linear (c) quadratic (d) cubic

ASSERTION - REASON BASED QUESTIONS

In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then

Assertion (A) : $f(x)$ has a minimum at $x = 1$.

Reason (R) : When $\frac{d}{dx}(f(x)) < 0, \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (a, a+h)$; where '$h$' is an infinitesimally small positive quantity , then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

Q20. **Assertion (A) :** The relation $f : \{1,2,3,4\} \to \{x, y, z, p\}$ defined by $f = \{(1,x),(2,y),(3,z)\}$ is a bijective function .

Reason (R) : The function $f : \{1,2,3\} \to \{x, y, z, p\}$ such that $f = \{(1,x),(2,y),(3,z)\}$ is one-one .

SECTION - B

Q21. Show that the elements on the main diagonal of a skew-symmetric matrix are all zero.

Q22. Differentiate w.r.t. x :$\sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

Q23. Evaluate : $\int \sin^5 x \ dx$

Q24. Find the equation of the curve passing through the point (1,1) whose differential equa tion is $x\,dy = (2x^2+1)\,dx \quad (x \neq 0)$

Q25. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$ where $\vec{a} = 2\hat{j} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

SECTION - C

Q26. Differentiate $\tan^{-1}\left(\frac{x-1}{x+1}\right)$ with respect to $\sin^{-1}(3x-4x^3)$, if $-\frac{1}{2} < x < \frac{1}{2}$.

MATHS KHAZANA

Q27. Two cards are drawn succesively with replacement from well shuffled pack of 52 cards. Find the probability distribution of the number of aces.

Q28. Evaluate : $\int_{-\pi}^{\pi} \frac{x(1-\sin x)}{1+\cos^2 x}\,dx$ **OR** Evaluate : $\int_{0}^{3/2} |x\cos \pi x|\,dx$

Q29. Maximise $Z = x + y$, subject to $x - y \le -1, -x + y \le 0,\ x, y \ge 0$.

Q30. Solve : $\frac{dy}{dx} + 2y\ \tan x = \sin x$; $y = 0$ when $x = \frac{\pi}{3}$

Q31. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & if\ x < \frac{\pi}{2} \\ a & if\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2} & if\ x > \frac{\pi}{2} \end{cases}$. If *f* is continuous at $x = \frac{\pi}{2}$, find *a* and *b*.

SECTION - D

Q32. Using integration, find the area of the triangular region whose sides have equations: $y = 2x + 1,\ y = 3x + 1$ and $x = 4$

Q33. Let N be the set of all natural numbers and let R be a relation on N × N, defined by $(a, b)\ R\ (c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in N \times N$. Show that R is an equivalence relation on N × N.

Q34. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Q35. Given $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, Compute $(AB)^{-1}$.

SECTION - E

Q36. In an examination, it is observed that 40% of the students failed in Chemistry, 30% failed in Physics and 15% failed in both Chemistry and Physics. A student is selected at random from the school.Use the given information to answer the following.

MATHS KHAZANA

(i) Find the probability that the selected student has failed in Chemistry, if it is known that he has failed in Physics.

(ii) Find the probability that the selected student has failed in Physics, if it is known that he has failed in Chemistry.

(iii) Find the probability that the selected student has failed in at least one of the two subjects.

(iv) Find the probability that the selected student has passed in at least one of the two ubjects.

Q37. Rahul and Harshit are playing a game using an unbiased die and a coin (having digits 1 on one face and 3 on the other face). On throwing the die, the number appeared belong to $\{1,2,3,4,5,6\}$. If A is a set of outcomes on tossing the coin *i.e.* $A=\{1,3\}$ and B be the set of all outcomes on throwing the die *i.e.* $B=\{1,2,3,4,5,6\}$.

Using given information answer the followings :

(i) Let $R: B \to B$ be defined by $R: \{(x,y): x-y \text{ is even integers}\}$. Is R symmetric ?

(ii) Rahul wants to know the number of relations from A to B. How many relations are possible ?

(iii) Rahul wants to know the number of functions from A to B. How many functions are possible ?

(iv) Harshit wants to know the number of bijective functions on B . How many bijections on B are possible ?

Q38. A volleyball player serves the ball which takes a parabolic path given by the equation

$h(t)=-\frac{7t^2}{2}+\frac{13t}{2}+1$, where $h(t)$ is the height of ball at any time t (in seconds), $(t \geq 0)$.

Based on the above information, answer the following questions:

(i) Is $h(t)$ a continuous function ? Justify.

(ii) Find the time at which the height of the ball is maximum.

MATHS KHAZANA

(a) Δ CDE (b) Δ ABC

(c) Quadrilateral OAEC (d) unbounded region

(iii) Intersection point of lines $2x + y = 10$ and $x + y = 6$ is

(a) $(3,4)$ (b) $(5,0)$ (c) $(3,3)$ (d) $(4,2)$

(iv) Maximum value of objective function

(a) 15 (b) 12 (c) 16 (d) 20

(v) If objective function is $Z = x + 2y$ then the maximum value of Z

(a) 8 (b) 12 (c) 16 (d) 20

MATHS KHAZANA

SAMPLE PAPER 14

SECTION - A

Q1. Let $n(\text{A}) = 3$ $n(\text{B}) = 3$ the numbers of surjective functions from A into B

(a) 3 (b) 6 (c) 12 (d) 24

Q2. The matrix having six elements in total with each element either 1 or 3. How many such matrices are possible ?

(a) 64 (b) 128 (c) 256 (d) 512

Q3. Let A be a non-singular matrix of order 3×3 such that $|\text{A}| = 4$ then the value of $\left| adj\left(\text{A}^{\text{T}}\right) \right|$.

(a) 4 (b) 8 (c) 16 (d) 32

Q4. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$ then $xy + yz + zx + xyz$ is :-

(a) -3 (b) 1 (c) 0 (d) 2

Q5. The value of $f(0)$ so that $f(x) = \dfrac{e^{2x} - 3^{2x}}{x}$ may be continuous at $x = 0$ is :

(a) $2\log\left(\dfrac{3}{e}\right)$ (b) $\log\dfrac{e}{3}$ (c) $2\log\dfrac{e}{3}$ (d) None of these

Q6. The function $f(x) = \sin\left(2x + \dfrac{\pi}{4}\right)$ is increasing on :

(a) $\left(\dfrac{3\pi}{8}, \dfrac{5\pi}{8}\right)$ (b) $\left(\dfrac{\pi}{8}, \dfrac{5\pi}{8}\right)$ (c) $\left(\dfrac{5\pi}{8}, \dfrac{7\pi}{8}\right)$ (d) Can't be determined

Q7. Value of : $\int \tan^5 x \sec^4 x \, dx$

(a) $\dfrac{\tan^5 x}{5} + \dfrac{\tan^7 x}{7} + c$ (b) $\dfrac{\sec^5 x}{5} + \dfrac{\sec^7 x}{7} + c$

(c) $\dfrac{\tan^6 x}{6} + \dfrac{\tan^8 x}{8} + c$ (d) $\dfrac{\sec^6 x}{6} + \dfrac{\sec^8 x}{8} + c$

Q8. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(a) $a = \dfrac{1}{\lambda}$ (b) $a = \dfrac{1}{|\lambda|}$ (c) $a = |\lambda|$ (d) None of these

Q9. The integrating factor of $\dfrac{dy}{dx} = y \sin x$ is

(a) $e^{\sin x}$ (b) $e^{-\sin x}$ (c) $e^{\cos x}$ (d) $e^{-\cos x}$

Q10. If $x = a\cos^3\theta$, $y = b\sin^3\theta$, then $\frac{dy}{dx}$ is :

(a) $-\frac{a}{b}\tan\theta$ (b) $\frac{a}{b}\cot\theta$ (c) $-\frac{b}{a}\tan\theta$ (d) $-\frac{b}{a}\cot\theta$

Q11. The angle between the lines $\vec{r} = 3\hat{i} - 2\hat{j} + 6\hat{k} + \lambda\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ and $\vec{r} = 2\hat{i} - 5\hat{j} + \mu\left(6\hat{i} + 3\hat{j} + 2\hat{k}\right)$ is :

(a) $\cos^{-1}\left(\frac{19}{29}\right)$ (b) $\cos^{-1}\left(\frac{19}{12}\right)$

(c) $\cos^{-1}\left(\frac{91}{21}\right)$ (d) $\cos^{-1}\left(\frac{19}{21}\right)$

Q12. Value of : $\int_{-2}^{2} \frac{3x^5}{7 + x^2}\, dx$

(a) 0 (b) $\frac{2}{3}$ (c) $\frac{4}{5}$ (d) $\frac{7}{11}$

Q13. A vector of magnitude 6 which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$ is :

(a) $-2\hat{i} + 4\hat{j} - 4\hat{k}$ (b) $-2\hat{i} + 4\hat{j} + 4\hat{k}$

(c) $-2\hat{i} - 4\hat{j} + 4\hat{k}$ (d) $-2\hat{i} - 4\hat{j} - 4\hat{k}$

Q14. The point of inflexion of $f\left(x\right) = 3x^4 - 4x^3 + 5$ in $\left[-2, 3\right]$ is

(a) -2 (b) 0 (c) 1 (d) 3

Q15. The range of real valued function $f(x) = -\sqrt{25 - x^2}$ is

(a) $\left[-5, 5\right]$ (b) $\left[0, 5\right]$ (c) $\left[-5, 0\right]$ (d) None

Q16. If $y = \cos^{-1}\left(\cos 5\right)$ then y is

(a) 5 (b) $\pi - 5$ (c) $2\pi - 5$ (d) $\pi + 5$

Q17. A bag contains 5 white and 3 black balls. 4 balls are successively drawn from the bag and not replaced, the probability that they are alternatively of different colours is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{7}$ (d) $\frac{4}{7}$

Q18. The solution set of the inequation $3x+5y<7$ is

(a) Whole XY - plane except the points lying on the line $3x+5y=7$.

(b) Whole XY - plane along with the points lying on the line $3x+5y=7$.

(c) open half plane containing the origin except the points of line $3x+5y=7$.

(d) open half plane not containing the origin.

ASSERTION - REASON BASED QUESTIONS

In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.

(b) Both A and R are true and R is not the correct expansion of A.

(c) A is true but R is false

(d) A is false but R is true .

Q19. **Assertion (A) :** The function $f:\mathrm{R}\to\mathrm{R}$ given by $f(x)=x^3$ in injective .

Reason (R) : The finction $f:\mathrm{X}\to\mathrm{Y}$ is injective , if $f(x)=f(y)\Rightarrow x=y, \forall x,y\in\mathrm{X}$.

Q20. **Assertion (A) :** If $\begin{vmatrix}2x & 5\\ 8 & x\end{vmatrix}=\begin{vmatrix}6 & -2\\ 7 & 3\end{vmatrix}$, then $=\pm 6$.

Reason (R) : If A and B are matrices of order 3 and $|\mathrm{A}|=4, |\mathrm{B}|=6$, then $|2\mathrm{AB}|=192$.

SECTION - B

Q21. If A and B are symmetric matrices of same orders then show that $\mathrm{AB}+\mathrm{BA}$ is also a symmetric matrix

Q22. Differentiate w.r.t. x : $\tan^{-1}\left(\dfrac{x}{\sqrt{a^2-x^2}}\right)$

Q23. Evaluate : $\displaystyle\int \frac{(x+1)e^x}{\cos^2(xe^x)}\,dx$

Q24. Solve : $\dfrac{dy}{dx}=\dfrac{1-\cos x}{1+\cos x}$

Q25. Prove that : $\left(\vec{a}\times\vec{b}\right)^2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{a}.\vec{b}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b}\end{vmatrix}$.

SECTION - C

Q26. Find $\dfrac{dy}{dx}$: $x=a\left(\cos t+\log\tan\dfrac{t}{2}\right),\quad y=a\sin t$.

Q27. The randon variable X has a probability distribution P (X) of the following form where k is some number.

$$P(X)=\begin{cases} k, & \text{if } x=0 \\ 2k, & \text{if } x=1 \\ 3k, & \text{if } x=2 \\ 0, & \text{otherwise} \end{cases}$$

(*a*) Determine k. (*b*) Find P (X < 2), P (X ≤ 2), and P (X ≥ 2)

Q28. Evaluate : $\int \frac{\sin x}{\sin 4x}\, dx$ **OR** Evaluate : $\int \frac{\sqrt{\cos 2x}}{\cos x}\, dx$

Q29. Solve the following problem graphically :
Minimise and Maximise $Z = 3x + 9y$
Subject : $x + 3y \le 60$, $x + y \ge 10$, $x \le y$, $x \ge 0, y \ge 0$

Q30. Solve : $\left(1+x^2\right)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y=0$ when $x=1$

Q31. If $f(x)$ is differentiable at $x = a$, find $\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x-a}$

SECTION - D

Q32. Using the method of integration find the area bounded by the curve $|x|+|y|=1$.

Q33. Let R be the relation on $N \times N$ defined by , $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$.
Check whether R is an equivalence relation $N \times N$.

Q34. Find the equation of the two lines through the origin which intersect line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$.

Q35. The cost of 4 *kg* onion, 3 *kg* wheat and 2 *kg* rice is Rs 60. The cost of 2 *kg* onion, 4 *kg* wheat and 6 *kg* rice is Rs 90. The cost of 6 *kg* onion 2 *kg* wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

SECTION - E

Q36. Rahul and Harshit are playing a game using an unbiased die and a coin (having digits 1 on one face and 3 on the other face). On throwing the die, the number appeared belong to $\{1,2,3,4,5,6\}$. If A is a set of outcomes on tossing the coin *i.e.* $A=\{1,3\}$ and B be the set of all outcomes on throwing the die *i.e.* $B=\{1,2,3,4,5,6\}$.

Using given information answer the followings :

(i) Let $R: B \rightarrow B$ be defined by $R:\{(x,y): x-y \text{ is even integers}\}$. Is R symmetric ?

(ii) Rahul wants to know the number of relations from A to B. How many relations are possible ?

(iii) Rahul wants to know the number of functions from A to B. How many functions are possible ?

(iv) Harshit wants to know the number of bijective functions on B . How many bijections on B are possible ?

Q37. Senior students tend to stay up all night and therefore are not able to wake up on time in morning Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who has 100% attendance attain A grade and10% irregular students attain A grade in their annual examination. At the end of the year, one students is chosen at random. from the class XII.

Using above information answer the followin.

(a) Find the conditional probability that a student attains A grade given that he is not 100% regular students.

(b) Find the probability of attaining A grade by the students of class XII.

(c) Find the probability that students is 100% regular given that he attains A grade.

(d) Find the probability that student is irregular given that he attains A grade.

Q38. Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.

Based on the above information, answer the following questions:

(i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A (x), the area of the garden.

(ii) Determine the maximum value of A (x)

SAMPLE PAPER 15

SECTION - A

Q1. If $f:\mathrm{R}\to \mathrm{A}$, given by $f(x)=x^2-2x+2$ is onto function, then the set A is

(a) $(1,\infty)$ (b) $[1,\infty)$ (c) $[2,\infty)$ (d) $(2,\infty)$

Q2. Let $\mathrm{A}=\left[a_{ij}\right]$ be a skew-symmetric matrix of order 3×3, the value of $a_{12}+a_{21}+a_{23}+a_{32}$

(a) $2\left(a_{12}+a_{23}\right)$ (b) $-2\left(a_{12}+a_{23}\right)$ (c) $\left(a_{12}-a_{23}\right)$ (d) 0

Q3. If A be a square matrix of order 3×3 such that $|\mathrm{A}|=-2$ then the value of $|adj(3\mathrm{A})|$.

(a) –729 (b) 729 (c) –2916 (d) 2916

Q4. The domain of $f(x)=\sin^{-1}x+\sin^2 x$

(a) $[0,1]$ (b) $[-1,0]$ (c) $(-1,1)$ (d) $[-1,1]$

Q5. The derivative of $f(x)=|x-1|+|x-5|$ at $x=3$ is :

(a) 3 (b) –3 (c) 0 (d) 1

Q6. The minimum value of $f(x)=3\sin x+4\cos x$

(a) –3 (b) –4 (c) –5 (d) –6

Q7. Value of : $\int_{-1/4}^{1/4}|\sin\pi x|\,dx$

(a) $\frac{2}{\pi}$ (b) 0 (c) $\frac{2}{\pi}\left(1-\frac{1}{\sqrt{2}}\right)$ (d) $\frac{2}{\pi}\left(1+\frac{1}{\sqrt{2}}\right)$

Q8. The angle between $\vec{a}$ and $\vec{b}$, if $\vec{a}.\vec{b}=3$ and $|\vec{a}\times\vec{b}|=3\sqrt{3}$, is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Q9. The integrating factor of $(x+1)\frac{dy}{dx}-y=e^x(x+1)^2$ is

(a) $(x+1)$ (b) $(x+1)^2$ (c) $\frac{1}{x+1}$ (d) $\frac{1}{(x+1)^2}$

Q10. If $y = \tan^{-1}\left(\frac{3-2x}{1+6x}\right)$, then $\frac{dy}{dx}$ is :

(a) $\frac{1}{1+4x^2}$ (b) $\frac{2}{1+4x^2}$ (c) $-\frac{2}{1+4x^2}$ (d) None of these

Q11. If the lines $\frac{1-x}{3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular , then k is equal to :

(a) $-\frac{10}{7}$ (b) $\frac{10}{7}$ (c) $\frac{7}{10}$ (d) $-\frac{7}{10}$

Q12. Solve : $\int \frac{1}{x\cos^2(1+\log x)}\,dx$

(a) $\sec(1+\log x)+c$ (b) $\sin(1+\log x)+c$

(c) $\tan(1+\log x)+c$ (d) $-\cot(1+\log x)+c$

Q13. The value of $\hat{i}.(\hat{j}\times\hat{k})+(\hat{i}\times\hat{k}).\hat{j}$ is :

(a) -1 (b) 0 (c) 1 (d) None of these

Q14. If the function $f(x)=3x^2-ax+b$ is increasing on $[1,5]$ then a lies in the interval.

(a) $(6,\infty)$ (b) $(-\infty,6)$ (c) $(-\infty,10)$ (d) $(10,\infty)$

Q15. Let $A=\{2,3,4\}$ and the relation R be defined on A as $R=\{(2,2),(3,4),(2,3)\}$. The minimum number of ordered pairs to be added in R to make it reflexive and transitive.

(a) 1 (b) 2 (c) 3 (d) 4

Q16. The value of $\tan^{-1}\frac{5}{4}-\tan^{-1}\frac{1}{9}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Q17. The letter of the word 'SOCIETY' are placed at random in a row, the probability that three vowels come together is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{14}$ (d) $\frac{1}{7}$

Q18. Which of the following points satisfies both the inequations $2x+y\le 10$ and $x+2y\ge 8$?

(a) $(-2,4)$ (b) $(3,2)$ (c) $(-5,6)$ (d) $(4,2)$

ASSERTION - REASON BASED QUESTIONS

In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. **Assertion (A) :** The function $f : R^* \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto , where R* is the set of all non-zero real numbers .

Reason (R) : The funcation $g : N \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto .

Q20. **Assertion (A) :** If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then $x = 2, y = 2, z = -5$ and $w = 4$.

Reason (R) : Two matrices are equal , if their orders are same and their corresponding elements are equal .

SECTION - B

Q21. The bookshop of a particular school has 10 *dozen* Chemistry books, 8 *dozen* Physics books, 10 *dozen* Economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Q22. If $y = x^{x^{x^{\dots\infty}}}$, find $\frac{dy}{dx}$.

Q23. Evaluate : $\int \frac{\sec x}{\sec 2x} dx$

Q24. Solve : $\frac{dy}{dx} + 3y = e^{-2x}$

Q25. Evaluate : $\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$

SECTION - C

Q26. If $(x-y)e^{\frac{x}{x-y}} = a$, prove that $y\frac{dy}{dx} + x = 2y$

Q27. A man wins a rupee for head and loses a rupee for tail when coin is tossed. Supposed that he tosses once and quits if he wins but tries once more if he loses on the first toss. Find the probability distribution of the number of rupees the man wins.

Q28. Evaluate : $\int \frac{\sin(x-a)}{\sin(x+a)} dx$ **OR** Evaluate : $\int \frac{\sec x}{1+\operatorname{cosec} x} dx$

Q29. Determine graphically the minimum value of the objective function $Z = -50x + 20y$

Subject : $2x - y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$, $x \geq 0$, $y \geq 0$

Q30. Solve : $\frac{dy}{dx} - 3y \cot x = \sin 2x; y = 2$ when $x = \frac{\pi}{2}$

Q31. If $x = \sec\theta - \cos\theta$, $y = \sec^n\theta - \cos^n\theta$, show that $(x^2 + 4)\, y_1^{\,2} = n^2(y^2 + 4)$.

SECTION - D

Q32. Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

Q33. Show that the function $f : \mathbf{R} \to \{x \in \mathbf{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$ is one-one and onto function.

Q34. A line makes angles α, β, γ and δ with the diagonals of a cube, prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

Q35. A company produces three products everyday. Their total production on a certain day is 45 *tons.* It is found that the production of third product exceeds the production of first product by 8 *tons* while the total production of first and third product is twice the production of second product. Determine the production level of each product using matrix method.

SECTION - E

Q36. One bag contains 4 yellow and 5 red balls. Another bag contains 6 yellow and 3 red balls. A ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag.

(i) If the ball drawn from second beg is yellow , what is the probability of tranferred ball being red ball ?

(ii) Find the probability that ball drawn is yellow.

Q37. Akansha visited the Exhibition along with her family, The Exhibition had a huge swing, which attracted many children. Akansha found that the swing traced the path of a parabola as given by $y = x^2$.

Answer the following questions using the above information.

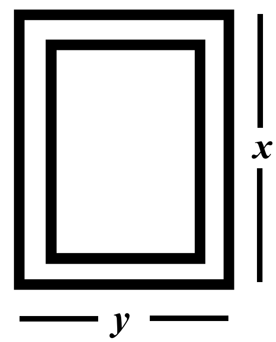
(i) Let $f : \mathrm{R} \rightarrow \mathrm{R}$ be defined by $f(x) = x^2$. Check bijectivity of $f(x)$.

(ii) Let $f : \mathrm{N} \rightarrow \mathrm{N}$ be defined by $f(x) = x^2$. Check injectivity of $f(x)$.

(iii) Let : $\mathrm{N} \rightarrow \mathrm{R}$ be defined by $f(x) = x^2$, what is the range of $f(x)$. **(2 + 1 + 1 = 4)**

Q38. There is a local printing press, whose owner is given a bulk order for printing of a magazine by a school of the same locality. He shows variety of pages to school administration. Follow ing is the pictorial description for a particular page, selected by school administration.

The total area of the page is $150\ \mathrm{cm}^2$. The combined width of the margin at the top and bottom is 3 cm and the side 2 cm. On the basis of above information answer the following questions :



(i) What is the relation between x and y ?

(ii) What is the area of the printable region of the page, in terms of x ?

(iii) What should be dimension of the page so that it has maximum area to be printed ? What is the maximum area ?

(1 + 1 + 2 = 4)

SAMPLE PAPER 16

SECTION A

Q1. If the area of the triangle with vertices $(-3,0),(3,0)$ and $(0,k)$ is 9 sq units , then the value's of k will be

(a) 9 (b) ± 3 (c) -9 (d) 6

Q2. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\frac{d^2y}{dx^2}$ is

(a) 4 (b) $\frac{3}{2}$ (c) 2 (d) Not defined

Q3. If the matrix $A=\begin{bmatrix}0 & a & -2\\ 3 & b & c\\ d & -4 & 0\end{bmatrix}$ is skew- symmetric, then value of $\frac{a+b}{c+d}$ is :

(a) -2 (b) 0 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Q4. If $f(x)=\begin{cases} x|x| & x\neq 0\\ 2k+1 & x=0\end{cases}$ is continuous at $x=0$, then the value of k is

(a) 0 (b) 1 (c) -1 (d) $-\frac{1}{2}$

Q5. If A and B are invertible square matrices of the same order , then which of the following is not correct ?

(a) $\text{adj } A=|A|\cdot A^{-1}$ (b) $\det(A)^{-1}=[\det(A)]^{-1}$

(c) $(AB)^{-1}=B^{-1}A^{-1}$ (d) $(A+B)^{-1}=B^{-1}+A^{-1}$

Q6. The lines $\vec{r}=\hat{i}+\hat{j}-\hat{k}+\lambda(2\hat{i}+3\hat{j}-6\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(6\hat{i}+9\hat{j}-18\hat{k})$; (where λ and μ are scalars) are

(a) coincident (b) perpendicular (c) parallel (d) intersecting

Q7. The corner points of the bounded feasible region determined by a system of linear constraints are (0, 3) , (1, 1) and (3, 0) . Let $Z=px+qy$, where $p,q>0$. The condition on p and q , so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) $p=2q$ (b) $p=\frac{q}{2}$ (c) $p=3q$ (d) $p=q$

Q8. ABCD is a rhombus whose diagonals intersect at E. Then, $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals to

(a) $\vec{0}$ (b) $\overrightarrow{AD}$ (c) $2\overrightarrow{BD}$ (d) $2\overrightarrow{AD}$

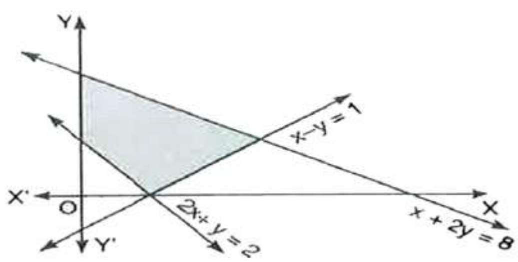
Q9. If $\int x^3\cos^6(x^4)\sin(x^4)dx = a\cos^7(x^4)+C$, then a is equal to :

(a) $-\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{28}$ (d) $\frac{1}{28}$

Q10. The value of $|A|$, if $A=\begin{bmatrix}0 & 3x-2 & \sqrt{y}\\ 2-3x & 0 & 2\sqrt{x}\\ -\sqrt{y} & -2\sqrt{x} & 0\end{bmatrix}$, where $x, y \in R^+$, is

(a) $(3x-2)^2$ (b) $(3x+2)y$ (c) $(3x-2)^2 y^{\frac{3}{2}}$ (d) 0

Q11. The feasible region corresponding to the linear constraints of a linear Programming Problem is given below .



Which of the following is not a constraint to the given linear Programming Problem ?

(a) $2x+y\geq 2$ (b) $x+2y\leq 8$ (c) $x-y>1$ (d) $x-y\leq 1$

Q12. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is

(a) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (b) $\frac{18}{25}(3\hat{i}+4\hat{k})$ (c) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (d) $\frac{18}{25}(2\hat{i}+4\hat{j})$

Q13. Given that A is a square matrix of order 3 and $|A|=-2$, then $|\text{adj}(2A)|$ is equal to

(a) -2^6 (b) 4 (c) -2^8 (d) 2^8

Q14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$, respectively . If the events of their solving the problem are independent , then the probability that the problem will be solved , is

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Q15. The general solution of the differential equation $ydx - xdy = 0$; (given $x, y > 0$), is of the form

(a) $xy = c$ (b) $x = cy^2$ (c) $y = cx$ (d) $y = cx^2$

(where 'c' is an arbitrary positive constant of integration)

Q16. The value of λ for which two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} - \lambda\hat{j} + \hat{k}$ are perpendicular is

(a) -2 (b) -4 (c) -6 (d) -8

Q17. The set of all points , where the function $f(x) = x + |x|$ is differentiable , is

(a) $(0, \infty)$ (b) $(-\infty, 0)$

(c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

Q18. If the direction cosines of a line are $< \frac{1}{c}, \frac{1}{c}, \frac{1}{c} >$, then

(a) $0 < c < 1$ (b) $c > 2$ (c) $c = \pm\sqrt{2}$ (d) $c = \pm\sqrt{3}$

ASSERTION - REASON BASED QUESTIONS

Directions (Q. Nos . 19-20) In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}(f(x)) = (x-1)^3(x-3)^2$, then

Assertion (A) : $f(x)$ has a minimum at $x = 1$.

Reason (R) : When $\frac{d}{dx}(f(x)) < 0, \forall x \in (a-h, a)$ and $\frac{d}{dx}(f(x)) > 0, \forall x \in (a+h, a)$; where '$h$' is an infinitesimally small positive quantity , then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

Q20. **Assertion (A) :** The relation $f : \{1, 2, 3, 4\} \to \{a, b, c, d\}$ defined by $f = \{(1, a), (2, b), (3, c)\}$ is a bijective function .

Reason (R) : The function $f : \{1, 2, 3\} \to \{a, b, c, d\}$ such that $f = \{(1, a), (2, b), (3, c)\}$ is one-one .

SECTION B

Q21. Find the value of $\tan\left[\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right]$.

OR

Find the domain of $\cos^{-1}\left(x^2-4\right)$.

Q22. Find the intervals in which the function $f:\mathrm{R}\to\mathrm{R}$ defined by $f(x)=x^2e^x$, is increasing.

Q23. If $f(x)=\frac{1}{x^2+4x+1}, x\in\mathrm{R}$, then find the maximum value of $f(x)$.

OR

Find the two numbers whose sum is 24 and whose product is as large as possible.

Q24. Evaluate $\int_{-1}^{1}\log_2\left(\frac{5-x^3}{5+x^3}\right)dx$.

Q25. Check whether the function $f:\mathrm{R}\to\mathrm{R}$ defined by $f(x)=x^3+x$, has any critical points or or not ? If yes , then find the points.

SECTION C

Q26. Evaluate $\int\frac{3x^2+2}{x^2\left(x^2+5\right)}dx; x\neq 0$.

Q27. The random variable X has a probability distribution P(X) of the following form , where '*k*' is some real number . $\mathrm{P}(\mathrm{X})=\begin{cases}k, & \text{if } \mathrm{X}=0\\ 2k, & \text{if } \mathrm{X}=1\\ 3k, & \text{if } \mathrm{X}=2\\ 0, & \text{otherwise}\end{cases}$

(i) Determine the value of k. (ii) Find $\mathrm{P}(\mathrm{X}<2)$. (iii) Find $\mathrm{P}(\mathrm{X}>2)$.

Q28. Evaluate $\int\sqrt{\frac{x}{1-x^3}}\,dx; x\in(0,1)$. **OR** $\int_0^{\pi/2}(2\log\sin x-\log\sin 2x)\,dx$

Q29. Solve the differential equation $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy, (y\neq 0)$.

OR

Solve the differential equation $\left(\cos^2 x\right)\frac{dy}{dx}+y=\tan x;\left(0\le x<\frac{\pi}{2}\right)$.

Q30. Solve the following linear programming problem graphically minimize , $Z=x+2y$.
Subject to the constraints , $x+2y\ge 100, 2x-y\le 0$, $2x+y\le 200, x, y\ge 0$.

OR

Solve the following linear programming problem graphically maximize , $Z=-x+2y$, subject to the constraints , $x\ge 3, x+y\ge 5$, $x+2y\ge 6, y\ge 0$.

Q31. If $(a+bx)e^{\frac{y}{x}}=x$, then prove that $x\frac{d^2y}{dx^2}=\left(\frac{a}{a+bx}\right)^2$.

SECTION D

Q32. Make a rough sketch of the region $\left\{(x,y):0\le y\le x^2+1, 0\le y\le x+1, 0\le x\le 2\right\}$ and find the area of the region , using the method of integration .

Q33. Let N be the set of all natural numbers and R be a relation on $\mathrm{N}\times\mathrm{N}$ defined by $(a,b)\,\mathrm{R}\,(c,d)\Leftrightarrow ad=bc$ for all $(a,b),(c,d)\in\mathrm{N}\times\mathrm{N}$. Show that R is an equivalence relation on $\mathrm{N}\times\mathrm{N}$. Also , find the equivalence class of (2, 6) i.e. [(2, 6)] .

OR

Show that the function $f:\mathrm{R}\to\left\{x\in\mathrm{R}:-1<x<1\right\}$ defined by $f(x)=\frac{x}{1+|x|}, x\in\mathrm{R}$ is one-one and onto function .

Q34. Using the matrix method , solve the following system of linear equations.
$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$.

Q35. Find the coordinates of the image of the point (1, 6, 3) with respect to the line $\vec{r}=\left(\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$, where ' λ ' is a scalar . Also , find the distance of the image from the Y-axis .

OR

An aeroplane is flying along the line $\vec{r}=\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$, where ' λ ' is a scalar and another aeroplane is flying along the line $\vec{r}=\hat{i}-\hat{j}+\mu\left(-2\hat{j}+\hat{k}\right)$, where ' μ ' is a scalar . At what point on the lines should they reach , so that the distance between them is the shortest ? Find the shortest possible distance between them .

SECTION E

Q36. Read the following passage and answer the questions given below .

In an office three employees James , Sophia and Oliver process incoming copies of a certain form .James processes 50% of the forms , Sophia processes 20% and Oliver the remaining 30% of the forms . James has an error rate of 0.06 , Sophia has an error rate of 0.04 and oliver has an error rate of 0.03 .

Based on the above information , answer the following questions .

(i) Find the probability that Sophia processed the form and committed an error .

(ii) Find the total probability of committing an error in processing the form .

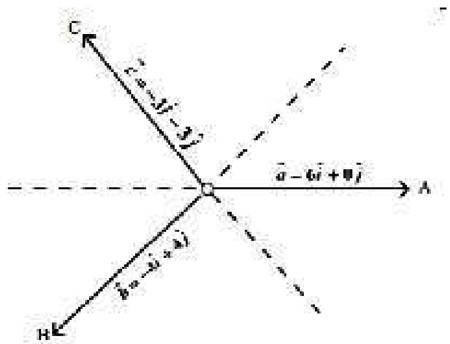
(iii) The manager of the Company wants to do a quality check . During inspection , he selects a form at random from the days output of processed form . If the form selected at random has an error , find the probability that the form is not processed by James .

OR

Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that James , Sophia and Oliver processed the form . Find the value of $\sum_{i=1}^{3} P(E_i/E)$.

Q37. Read the following passage and answer the questions given below .

Teams A, B , C went for playing a tug of war game . Team A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area .Team A pulls with force $F_1 = 6\hat{i} + 0\hat{j}\ k\text{N}$ Team B pulls with force $F_2 = -4\hat{i} + 4\hat{j}\ k\text{N}$,Team C pulls with force $F_3 = -3\hat{i} - 3\hat{j}\ k\text{N}$,



(i) What is the magnitude of the force of Team A?

(ii) Which team will win the game ?

(iii) Find the magnitude of the resultant force exerted by the teams .

OR

In what direction is the ring getting pulled ?

Q38. Read the following passage and answer the questions given below .

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight , for $x \le 3$.

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight .

(ii) Does the rate of growth of the plant increase or decrease in the first three days ? What will be the height of the plant after 2 days ?

SAMPLE PAPER 17

SECTION A

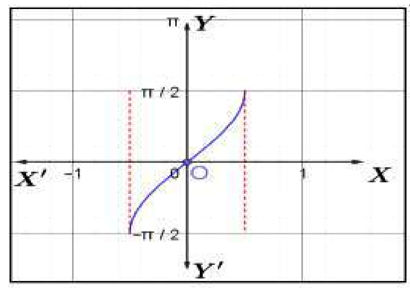
Q1. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then the value of $[A+2B]'$ is

(a) $\begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} -4 & 5 \\ 6 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 1 \\ 2 & 4 \end{bmatrix}$

Q2. The interval in which $y = x^2 e^{-x}$ is increasing with respect to x is

(a) $[0,2]$ (b) $(0,2)$ (c) $[0,2)$ (d) $(0,2]$

Q3. Identify the function shown in the graph :



(a) $\sin^{-1} x$ (b) $\sin^{-1}(2x)$

(c) $\sin^{-1}\left(\frac{x}{2}\right)$ (d) $2\sin^{-1} x$

Q4. A drone flies through a distance in a straight line given by the vector $3\hat{i} - 2\hat{j} + \hat{k}$. A man stand ing beside a straight road given by $\vec{r} = (3+\lambda)\hat{i} + (2\lambda - 1)\hat{j} + 3\lambda\hat{k}$ is observing the drone. The projected length of drone flight on the road is :

(a) $\frac{14}{\sqrt{6}}$ units (b) $\frac{1}{\sqrt{14}}$ units (c) $\frac{2}{\sqrt{14}}$ units (d) $\frac{7}{\sqrt{6}}$ units

Q5. If A and B are symmetric matrices of same order , then $(AB' - BA')$ is a

(a) symmetric matrix (b) skew-symmetric matrix
(c) identify matrix (d) null matrix

Q6. If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} A = \begin{bmatrix} 2 & 1 & 3 & 7 \end{bmatrix}$, then the order of matrix A is

(a) 2×4 (b) 4×2 (c) 4×3 (d) 3×4

Q7. The set of all points , where the function $f(x) = x|x|$ is differentiable , is

(a) $(0, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, 0) \cup (0, \infty)$ (d) $(-\infty, \infty)$

Q8. If A is a square matrix of order 4 and |adj A| = -125, then A (adj A) is equal to :

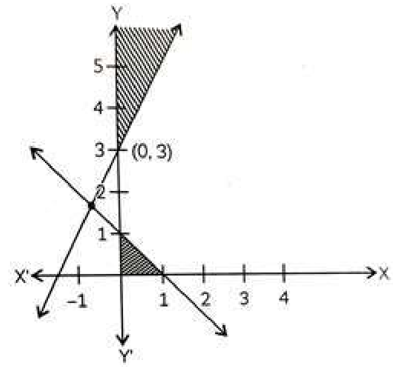
(a) 5 (b) -5 (c) 5I (d) -5I

Q9. If $\frac{d}{dx}\left(f\left(x\right)\right)=\log x$, then $f\left(x\right)$ equals :

(a) $-\frac{1}{x}+C$ (b) $x\left(\log x-1\right)+C$ (c) $x\left(\log x+x\right)+C$ (d) $\frac{1}{x}+C$

Q10. Shown below is a graph for an LPP. Which of the following is true about the LPP ?

(a) Feasible region is bounded.
(b) Feasible region is unbounded.
(c) Every solution is infeasible solution.
(d) Cannot conclude anything for given LPP.



Q11. If $\int \sec^{2n+1} x \tan x \, dx = \frac{\sec^5 x}{5}+c$ then the value of n is

(a) 1 (b) 2 (c) 3 (d) 4

Q12. The minimum value of function $f\left(x\right)=\left|x+2\right|-1$ is

(a) 1 (b) 0 (c) -1 (d) -2

Q13. If $\begin{bmatrix} 3y-x & -2x \\ 3 & 7 \end{bmatrix}=\begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, then the value of $x+y$ is

(a) 1 (b) 2 (c) 5 (d) 3

Q14. The direction cosines of a line which makes equal angles with the coordinate axes are

(a) $\pm\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (b) $\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

(c) $\pm\left(\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$ (d) $\pm\left(1,1,1\right)$

Q15. If two line $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $r=\vec{a}_2+\mu\,\vec{b}_2$ are coplanar then the value of $\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)$ is

(a) 1 (b) 2 (c) 0 (d) -1

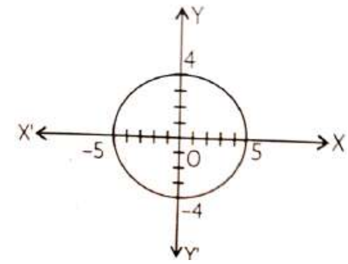
Q16. For what value of a the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}+6\hat{j}-8\hat{k}$ are collinear ?

(a) 1 (b) 2 (c) -3 (d) -4

Q17. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then the range of $|\lambda \vec{a}|$ is

(a) $[2,8]$ (b) $[-12,8]$ (c) $[-12,8)$ (d) $(-12,8)$

Q18. Choose the correct option :



(a) The area of the shaded region is 20π sq. units
(b) The area of the shaded region is 24π sq. units
(c) The area of the shaded region is 16π sq. units
(d) The area of the shaded region is 25π sq. units

ASSERTION - REASON BASED QUESTIONS

Directions (Q. Nos . 19-20) In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. **Assertion (A) :** The function $f : \mathrm{R} \to \mathrm{R}$ given by $f(x) = x^3$ in injective .

Reason (R) : The finction $f : \mathrm{X} \to \mathrm{Y}$ is injective , if $f(x) = f(y) \Rightarrow x = y, \forall x, y \in \mathrm{X}$.

Q20. **Assertion (A) :** If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then $= \pm 6$.

Reason (R) : If A and B are matrices of order 3 and $|\mathrm{A}| = 4, |\mathrm{B}| = 6$, then $|2\mathrm{AB}| = 192$.

SECTION B

Q21. If A and B are two symmetric matrices of same order . Then , find whether the matrix $\mathrm{AB} - \mathrm{BA}$ is symmetric or skew-symmetric .

OR

If A is a 3×3 non-singular matrix such that $\mathrm{AA^T} = \mathrm{A^TA}$ and $\mathrm{B} = \mathrm{A^{-1}A^T}$, then what will be $\mathrm{BB^T}$?

Q22. If $y = \log\left[\sin\left(x^2\right)\right], 0 < x < \frac{\pi}{2}$, then find $\frac{dy}{dx}$ at $x = \frac{\sqrt{\pi}}{2}$.

Q23. If $\vec{a}$ and $\vec{b}$ are unit vectors , then find the angle between $\vec{a}$ and $\vec{b}$, given that $\left(\sqrt{3}\vec{a} - \vec{b}\right)$ is a unit vector .

OR

The x-coordinate of a point on the line joining the points $\mathrm{P}(2,2,1)$ and $\mathrm{Q}(5,1,-2)$ is 4 . Find its z-coordinate .

Q24. Find the angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{1}=\frac{z+5}{1}$.

Q25. Find the solution of equation $(2y-1)dx-(2x+3)dy=0$.

SECTION C

Q26. Evaluate $\int \frac{5x^2+3}{x^2\left(x^2+4\right)}dx$. **OR** Find $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$.

Q27. Find the area bounded by the curve $y^2=4ax$ and the line $y=2a$ and Y-axis .

Q28. If $f:\mathrm{R}-\{2\}\to\mathrm{R}-\{3\}$ is defined by $f(x)=\frac{3x+1}{x-2}$, where R is the set of real numbers , show that *f* is bijective .

Q29. Solve the differential $(x\,dy-y\,dx)\,y\sin\left(\frac{y}{x}\right)-(y\,dx+x\,dy)\,x\cos\left(\frac{y}{x}\right)=0$.

OR

Solve the differential $\left(x^2-yx^2\right)dy+\left(y^2+x^2y^2\right)dx=0$, given that $y=1$, when $x=1$.

Q30. Find the domain of the function $f(x)=\cos^{-1}x+\sin^{-1}2x$.

Q31. If $y=x^{x^2}$, find $\frac{dy}{dx}$ **OR** If $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$, find $\frac{dy}{dx}$.

SECTION D

Q32. Solve the given LPP graphically Minimise $Z=5x+8y$, subject to constraints $2x+y\ge 140,\ 3x+5y\ge 350,\ x,y\ge 0$

OR

Solve the given LPP graphically Maximise $Z=22x+18y$, subject to constraints $x+y\le 20$, $x+y\le 20,\ 360x+240y\le 5760,\ x,y\ge 0$.

Q33. Evaluate $\int_0^{\pi}\log(1+\cos x)\,dx$.

Q34. Find the area of the greatest isosceles triangle that can be inscribed in a ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having its vertex coincident with one end of the major axis.

Q35. Find the foot of the perpendicular drawn from the point $2\hat{i}-\hat{j}+5\hat{k}$ to the line $\vec{r}=\left(11\hat{i}-2\hat{j}-8\hat{k}\right)+\lambda\left(10\hat{i}-4\hat{j}-11\hat{k}\right)$. Also , find the length of the perpendicular .

OR

Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{\left|\vec{a}\times\vec{b}\right|}{2}$. Also , find the area of the parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

SECTION E

Q36. A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2-k$

On the basis of above information , answer the following questions .

(i) Find the value of $P(1 \le x < 3)$ in terms of k .

(ii) Find the value of $P(0 \le x < 4)$ in terms of k .

(iii) Find the value of k and then evaluate $P(x < 6)$.

OR

If $k = \frac{1}{10}$, evaluate $P(x \ge 6)$ and $P(0 < x < 5)$.

Q37. The sum of the surface area of a rectangular parallelopiped with sides $x, 2x$ and $\frac{x}{3}$ and a sphere of radius y is given to be constant .

On the basis of above information , answer the following questions .

(i) If S is the constant , then find the relation between S, x and y .

(ii) If the combined volume is denoted by V , then find the relation between V, x and y .

(iii) Find the relation between x and y when the volume V is minimum.

OR

If at $x = 3y$, volume V is minimum , then find the value of minimum volume and the value of S.

Q38. An insurance company believes that people can be divided into two classes : those who are accident prone and those who are not. The company's statistics show that an accident-prone person will have an accident at sometime within a fixed one-year period with probability 0.6 , whereas this probability is 0.2 for a person who is not accident prone . The company knows that 20% of the population is accident prone .

On the basis of above information , answer the following questions .

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy ?

(ii) Suppose that a new policy holder has an accident within a year of purchasing a policy . What is the probability that he or she is accident prone ?

SAMPLE PAPER 18

SECTION A

Q1. If $\int_0^1 \left(3x^2 + 2x + k\right) dx = 0$, then the value of k is

(a) 2 (b) 0 (c) 1 (d) -2

Q2. The value of λ, if the vector $2\hat{i} + \lambda\hat{j} - 4\hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$ are perpendicular is

(a) 0 (b) 1 (c) 2 (d) -1

Q3. If a relation R on the set $\{1, 2, 3, 4\}$ be defined by $R = \{(x, y) : y = 2x\}$, then R is

(a) reflexive (b) symmetric

(c) transitive (d) neither reflexive nor symmetric nor transitive

Q4. Set A has 4 elements and the set B has 5 elements. Then the number of injective mappings that can be defined from A to B is

(a) 24 (b) 60 (c) 120 (d) 210

Q5. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) $[1,2]$ (b) $(1,2)$ (c) $[1,2)$ (d) $(1,2]$

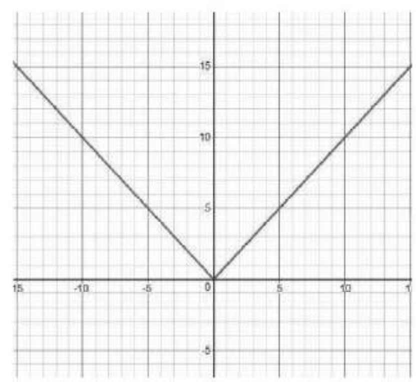
Q6. The function $f : R \to R$ shown in the graph is :

(a) one-one and onto

(b) one-one but not onto

(c) onto but not one-one

(d) neither one-one nor onto



Q7. For the matrices A and B, if multiplication is defined and AB = A and BA = B, then B^2 is :

(a) A (b) B (c) I (d) AB

Q8. A and B are events such that $P(A/B) = P(B/A)$ then

(a) $A \subset B$ (b) $B = A$ (c) $A \cap B = \phi$ (d) $P(A) = P(B)$

Q9. The number of reflexive relations are possible in a set A where $n(A) = 3$ are

(a) 64 (b) 32 (c) 128 (d) 1024

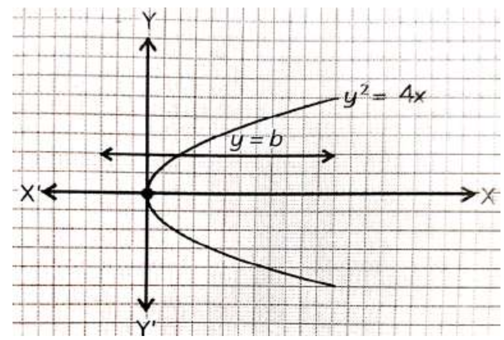
Q10. Let $A = \begin{bmatrix} 200 & 50 \\ 10 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 50 & 40 \\ 10 & 3 \end{bmatrix}$, then $|AB|$ is equal to

(a) 460 (b) 2000 (c) 3000 (d) -7000

Q11. Area (in sq. units) of the region bounded by the parabola, y-axis and the line $y = b$ is :

(a) $\frac{b^3}{24}$ (b) $\frac{b^2}{16}$

(c) $\frac{b^3}{8}$ (d) $\frac{b^3}{12}$



Q12. If $x = t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ is

(a) $\frac{3t}{4}$ (b) $\frac{3}{t}$ (c) $\frac{3}{4t}$ (d) $\frac{4}{3t}$

Q13. $\int e^x \left(1 - \cot x + \operatorname{cosec}^2 x\right) dx$ is equal to

(a) $e^x\left(1 - \cot x\right) + C$ (b) $e^x\left(1 + \cot x\right) + C$

(c) $e^x\left(\operatorname{cosec} x\right) + C$ (d) $e^x \cot x + C$

Q14. Direction ratio if a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are:

(a) $2, -1, 6$ (b) $2, 1, 6$ (c) $2, 1, 3$ (d) $2, -1, 3$

Q15. The vector $\vec{r}$ is inclined at an equal angles to the coordinate axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then $\vec{r}$ is

(a) $\pm 2\left(\hat{i} + \hat{j} + \hat{k}\right)$ (b) $2\left(\hat{i} + \hat{j} - \hat{k}\right)$

(c) $2\left(\hat{i} - \hat{j} - \hat{k}\right)$ (d) $2\left(\hat{i} - \hat{j} + \hat{k}\right)$

Q16. How many arbitrary constants are there in the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2; y(0) = 1$?

(a) 1 (b) 2 (c) 0 (d) 3

Q17. For what value of n is the following differential equation homogeneous $\frac{dy}{dx} = \frac{x^3 - y^n}{x^2y + xy^2}$?

(a) 1 (b) 2 (c) 3 (d) 4

Q18. $\int_1^3 [x]\,dx$ is equal to

(a) 0 (b) 1 (c) -1 (d) 2

ASSERTION - REASON BASED QUESTIONS

Directions (Q. Nos . 19-20) In the following questions , a statements of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the followinng choices .

(a) Both A and R are true and R is the correct expansion of A.
(b) Both A and R are true and R is not the correct expansion of A.
(c) A is true but R is false
(d) A is false but R is true .

Q19. **Assertion (A) :** The function $f: R^* \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto , where R* is the set of all non-zero real numbers .

Reason (R) : The funcation $g: N \to R^*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto .

Q20. **Assertion (A) :** If $\begin{bmatrix} xy & 4 \\ z+5 & x+y \end{bmatrix} = \begin{bmatrix} 4 & w \\ 0 & 4 \end{bmatrix}$, then $x = 2, y = 2, z = -5$ and $w = 4$.

Reason (R) : Two matrices are equal , if their orders are same and their corresponding elements are equal .

SECTION B

Q21. Find the points of discontinuity of $f(x) = \begin{cases} |x|+3 & , \text{ if } x \le -3 \\ -2x & , \text{ if } -3 < x < 3 \\ 6x+2 & , \text{ if } x \ge 3 \end{cases}$.

Q22. Evaluate $\int e^{2x^2 + \log x}\, dx$. **OR** Evaluate $\int \tan^8 x \sec^4 x\, dx$.

Q23. Find a 2×2 matrix B such that : $B\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$.

Q24. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Q25. Solve : $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

SECTION C

Q26. Consider the function $f : \mathrm{R}^+ \to [4,\infty)$ defined by $f(x) = x^2 + 4$, where R^+ is the set of all non-negative real numbers . Show that *f* is bijective .

Q27. Show that the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$ is homogeneous . Find the particular solution of this differential equation , given that $y = \frac{\pi}{4}$, when $x = 1$.

OR

Find the solution of differential equation $x^2dy + y(x+y)dx = 0$, if $x = 1$ and $y = 1$.

Q28. Evaluate : $\int \sqrt{\tan x} + \sqrt{\cot x}\,dx$. **OR** Evaluate : $\int \frac{1}{\sin x + \sin 2x}dx$.

Q29. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 1, |\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}$ and $\vec{c}$ are perpendicular to each other, find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

Q30. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing funcation of x, throughout its domain.

OR

Find the intervals in which the funcation given by $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$ is
(i) increasing. (ii) decreasing .

Q31. Evaluate : $\int \frac{x^4 + 1}{x^6 + 1}\,dx$.

SECTION D

Q32. If $\mathrm{A} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, then show that A satisfies the following equation. $\mathrm{A}^3 - 4\mathrm{A}^2 + 11\mathrm{I} - 3\mathrm{A} = \mathrm{O}$

OR

Solve using matrix method . $x + 3y + 4z = 8$, $2x + y + 2z = 5$, $5x + y + z = 7$

Q33. Find the length and the equations of the line of shortest distance between the lines .

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}.$$

Q34. Solve the given graphically Maximise $Z=7x+6y$, subject to constraints

$x+y\le 50,\ 2x+y\le 80,\ x,y\ge 0$

Q35. Evaluate $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$.

OR

Prove that $\int_0^{\pi} \frac{x}{(1+\sin x)}dx=\pi$.

SECTION E

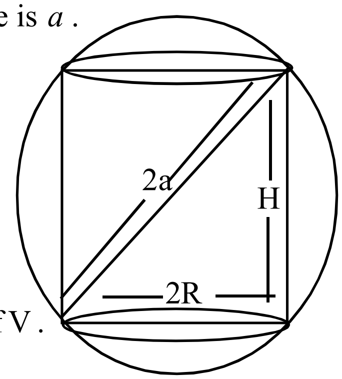
Q36. A curve passing through the point (0,1).If the slope of the tangent to the curve at any point (x,y) is equal to the sum of the x coordinate (abscissa) and the product of the x coordinate and y coordinate (ordinate)of that point. Answer the followings :

(i) What is the differential equation on basis of given information ?

(ii) Find the general solution of the differential equation.

(iii) Find the equation of curve. **(1 + 2 + 1 = 4)**

Q37. A toy making company made a toy in which a cylinder is inscribed in a sphere . The height and radius of cylinder is H and R, respectively , while the radius of sphere is a .



On the basis of above information , answer the following questions .

(i) Find the relation between R and H .

(ii) Find the value of volume of cylinder V in terms of H.

(iii) Find the value of H when V is maximum .

OR

If V is maximum at $H=\frac{2}{\sqrt{3}}a$, then find the maximum value of V .

Q38. A doctor is to visit a patient . From the past experience , it is known that the probabilities that he will come by train , bus, scooter or by other means of transport are respectively $\frac{3}{10},\frac{1}{5},\frac{1}{10}$ and $\frac{2}{5}$. The probability that he will be late are $\frac{1}{4},\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train , bus and scooter respectively, but if he comes by other means of transport , then he will not be late .

On the basis of above information , answer the following questions .

(i) Find the probability that doctor comes by train and he is late .

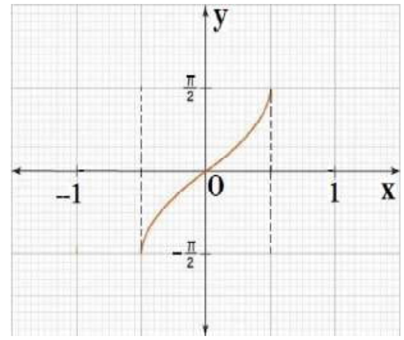
(ii) Find the probability that he is late.

(iii) Find the probability that he comes by scooter given that he is late .

SAMPLE PAPER 19

SECTION A

Q1. Identify the function shown in the graph.



(a) $\sin^{-1} x$ (b) $\sin^{-1}(2x)$

(c) $\sin^{-1}\left(\frac{x}{2}\right)$ (d) $2\sin^{-1} x$

Q2. If A is a square matrix of order 4 and $|adj\ \text{A}| = 27$, then A (adj A) is equal to

(a) 3 (b) 9 (c) 3I (d) 9I

Q3. If the matrix $\text{A} = \begin{bmatrix} 0 & a & -2 \\ 3 & b & c \\ d & -4 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $\frac{a+b+c}{d}$ is

(a) -2 (b) 0 (c) 1 (d) 2

Q4. Inverse trigonometric function, whose domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$, is

(a) $\sin^{-1} x$ (b) $\sin^{-1}\left(\frac{x}{3}\right)$ (c) $\sin^{-1}(3x)$ (d) $3\sin^{-1} x$

Q5. The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(a) $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 3 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (c) $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{4} \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

Q6. If a function defined by $f(x) = \begin{cases} kx+1, & x \le \pi \\ \cos x, & x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is

(a) π (b) $-\frac{1}{\pi}$ (c) 0 (d) $-\frac{2}{\pi}$

Q7. Value of the determinant $\begin{bmatrix} \cos 74° & \sin 74° \\ \sin 16° & \cos 16° \end{bmatrix}$ is

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

Q8. If $f(x) = x \sin^{-1} x$, then $f'\left(\frac{1}{2}\right)$ is equal to

(a) $\frac{\pi}{6} - \frac{1}{2}$ (b) $\frac{\pi}{6} + \frac{1}{2}$ (c) $-\frac{\pi}{6} - \frac{1}{2}$ (d) $\frac{\pi}{6} + \frac{1}{\sqrt{3}}$

Q9. A function $f(x) = 10 - x - 2x^2$ is increasing on the interval

(a) $\left(-\infty, -\frac{1}{4}\right]$ (b) $\left(-\infty, \frac{1}{4}\right)$ (c) $\left[-\frac{1}{4}, \infty\right)$ (d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

Q10. If $f(a+b-x) = f(x)$, then $\int_a^b x f(x)\, dx$ is equal to

(a) $\frac{a+b}{2} \int_a^b f(b-x)\, dx$ (b) $\frac{a+b}{2} \int_a^b f(a-x)\, dx$

(c) $\frac{b-a}{2} \int_a^b f(x)\, dx$ (d) $\frac{a+b}{2} \int_a^b f(x)\, dx$

Q11. The differential equation $\frac{dy}{dx} = \frac{x(1+y^2)}{y(1+x^2)}$ represents a family of :

(a) straight lines (b) ellipses (c) parabolas (d) hyperbolas

Q12. If $\int x^2 \tan^3(x^3) \sec^2(x^3)\, dx = a \tan^4(x^3) + C$, then a is equal to

(a) $-\frac{1}{10}$ (b) $\frac{1}{20}$ (c) $\frac{1}{12}$ (d) $\frac{1}{15}$

Q13. A bird flies through a distance in a straight line given by the vector $\hat{i} + 2\hat{j} + \hat{k}$. A man standing beside a straight metro rail track given by $\vec{r} = (3+\lambda)\hat{i} + (2\lambda - 1)\hat{j} + 3\lambda\hat{k}$ is observing the bird. The projected length of its flight on the metro track is

(a) $\frac{6}{\sqrt{14}}$ units (b) $\frac{14}{\sqrt{6}}$ units (c) $\frac{8}{\sqrt{14}}$ units (d) $\frac{5}{\sqrt{6}}$ units

Q14. The distance of the point with position vector $4\hat{i} - 3\hat{j} + 7\hat{k}$ from the z-axis is

(a) 7 units (b) 4 units (c) 5 units (d) $\sqrt{58}$ units

Q15. If $\vec{a} = 3\hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 3\hat{k}$ and $c = 6\hat{i} - \hat{j} + 2\hat{k}$ are three given vectors, then $\left(2\vec{a}.\hat{i}\right)\hat{i} - \left(\vec{b}.\hat{j}\right)\hat{j} + \left(\vec{c}.\hat{k}\right)\hat{k}$ is same as the vector

(a) $\vec{a}$ (b) $\vec{b} + \vec{c}$ (c) $\vec{a} - \vec{b}$ (d) $\vec{c}$

Q16. A student of class XII studying Mathematics comes across an incomplete question in a book.

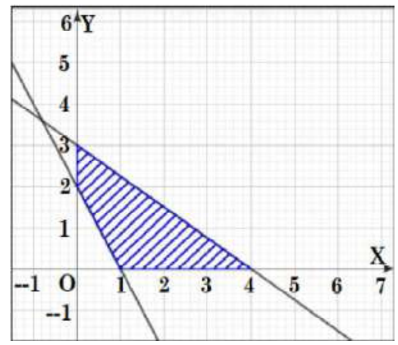
Maximize $Z = 3x + 2y + 1$

Subject to the constraints $x \geq 0,\ y \geq 0,\ 3x + 4y \leq 12$.

S/he notices the below shown graph for the said LPP, and finds that a constraint is missing in it.

Help her/him choose the required constraint from the graph.

The missing contraint is



(a) $x + 2y \leq 2$ (b) $2x + y \geq 2$ (c) $2x + y \leq 2$ (d) $x + 2y \geq 2$

Q17. If $Z = ax + by + c$, where $a, b, c > 0$, attains its maximum value at two of its corner points (4, 0) and (0, 3) of the feasible region determined by the system of linear inequalities, then

(a) $4a = 3b$ (b) $3a = 4b$ (c) $4a + c = 3b$ (d) $3a + c = 4b$

Q18. A person observed the first 4 digits of your 6-digit PIN. What is the probability that the person can guess your PIN?

(a) $\frac{1}{81}$ (b) $\frac{1}{100}$ (c) $\frac{1}{90}$ (d) 1

Assertion-Reason

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true .

Q19. **Assertion :** Value of the expression $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}\left(\sqrt{2}\right)$ is $\frac{\pi}{4}$.

Reason : Principal value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$.

Q20. **Assertion :** Given two non-zero vectors $\vec{a}$ and $\vec{b}$. If $\vec{r}$ is another non-zero vector such that $\vec{r} \times \left(\vec{a} + \vec{b}\right) = \vec{0}$, then $\vec{r}$ is perpendicular to $\vec{a} \times \vec{b}$.

Reason : The vector $\left(\vec{a}+\vec{b}\right)$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$.

Section - B

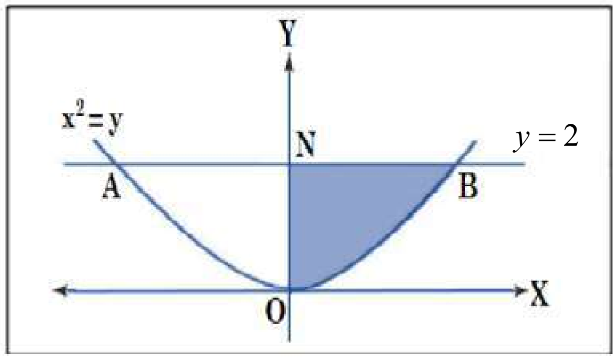
Q21. If $y = \log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then prove that $\frac{dy}{dx} - \sec x = 0$.

Q22. Solve : $\cos(2\sin^{-1}x) = \frac{1}{9}, x > 0$

Q23. Find : $\int \frac{(x-3)}{(x-1)^3} e^x \, dx$.

OR

Find out the area of shaded region in the enclosed figure.



Q24. If $f(x+y) = f(x)f(y)$ for all $x, y \in \text{R}$ and $f(5) = 2, f'(0) = 3$, then using the definition of derivatives, find $f'(5)$.

Q25. The two vectors $\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+3\hat{k}$ represent the two sides OA and OB, respectively of a ΔOAB, where O is the origin. The point P lies on AB such that OP is a median. Find the area of the parallelogram formed by the two adjacent sides as OA and OP.

Section - C

Q26. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$ and hence find its value at $x = e$.

OR

If $f(2) = 4$ and $f'(2) = 1$, then find $\lim_{x\to 2} \frac{x\ f(2) - 2f(x)}{x-2}$.

Q27. A spherical ball of ice melts in such a way that the rate at which its volume decreases at any instant is directly proportional to its surface area. Prove that the radius of the ice ball decreases at a constant rate.

Q28. Find the area under the curve $y=\sqrt{6x+4}$ above x- axis from $x=0$ to $x=2$. Draw a sketch of the curve also.

OR

Using integration find the area of the region $\{(x,y): x^2-4y\le 0,\ y-x\le 0\}$.

Q29. Find the distance of point $(2,-1,3)$ from the line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+\mu(3\hat{i}+6\hat{j}+2\hat{k})$ measured parallel to the z-axis.

OR

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ plane.

Q30. Solve the linear programming problem (L.P.P.) graphically.
Maximize $Z=2x+y$

Subject to $x+y\le 1200,\ x+y\ge 600,\ y\le \frac{x}{2},\ x\ge 0,\ y\ge 0$.

Q31. Two students Mehul and Rashi are seeking admission in a college. The probability that Mehul is selected is 0.4 and the probability of selection of exactly one of them is 0.5. Chances of selection of them is independent of each other. Find the chances of selection of Rashi. Also find the probability of selection of at least one of them.

Section - D

Q32. For two matrices $A=\begin{bmatrix}3 & -6 & -1\\ 2 & -5 & -1\\ -2 & 4 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2 & -1\\ 0 & -1 & -1\\ 2 & 0 & 3\end{bmatrix}$, find the product AB and hence solve the system of equations : $3x-6y-z=3,\ 2x-5y-z+2=0,\ -2x+4y+z=5$.

Q33. Evaluate : $\int_0^{\pi/2}\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}\,dx$ **OR** Find : $\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}\,d\theta$.

Q34. Solve the differential equation : $y+\frac{d}{dx}(xy)=x(\sin x+x)$.

OR

Find the particular solution of the differential equation : $2ye^{\frac{x}{y}}\,dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ given that $y(0)=1$.

Q35. The lines $\frac{x-1}{3} = -y,\ z+1=0$ and $-\frac{x}{2} = \frac{y+1}{2} = z+2$ intersect at a point whose y-coordinate is. Find the coordinates of their point of intersection. Find the vector equation of a line perpendicular to both the given lines and passing through this point of intersection.

<u>Section - E</u>

Q36. A company is analysing its internal communication system between the five departments labelled as P, Q, R, S and T. The company has gathered the following data about one-way communication between departments.

- Messages are sent from P to Q, P to R and P to S.
- Messages are sent from Q to R and Q to T.
- Messages are sent from R to T.
- Messages are sent from S to T and S to Q.

The company wants to represent this communication system using the concepts of relations and functions. Use the given data to answer the following questions.

i) Is the communication relation (X) reflexive? Justify.

ii) Is the communication relation (X) symmetric? Justify.

iii) a) Represent the communication relation (X) as a set of ordered pairs. Also, state the domain and range of this relation. Is the communication relation (X) transitive? Justify your answer.

OR

iii) b) Does the communication relation (X) represent a function? Justify your answer.

Q37. Given three identical boxes I, II and III , each containing two coins . In box I both coins are gold coins , in box II both are silver coins and in box III there is one gold and one silver coin. A person chooses a box at random and takes out a coin .

On the basis of above information , answer the following questions .

(i) Find the probability of choosing one box and the probability of getting gold coin from III box .

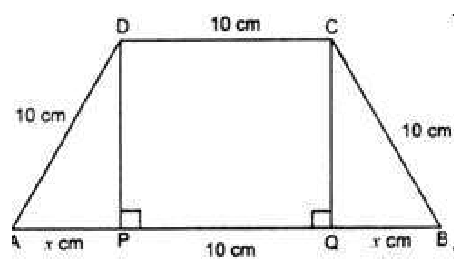
(ii) Find the probability of choosing III box and getting gold coin .

(iii) Find total probability of drawing gold coin .

OR

If drawn coin is of gold , then find the probability that other coin in box is also of gold .

Q38. A building has front gate of the shape as shown below . It is the shape of trapezium whose three sides other than base is 10m. Height of the gate is h cm.



On the basis of above information , answer the following questions .

(i) Find the relation between x and h .

(ii) Express the area of gate A in terms of x.

(iii) Find the value of x when the area A is maximum .

OR

If at $x = 5$ the area A is maximum , then find the value of h and the maximum value of A.

SAMPLE PAPER 20

SECTION A

Q1. For a square matrix A, if we have A. (adj.A) $= \begin{bmatrix} -1978 & 0 & 0 \\ 0 & -1978 & 0 \\ 0 & 0 & -1978 \end{bmatrix}$, then $|A| + |adj.A| =$

(a) $1978^2 \times 1977$ (b) -1977 (c) 1978×1977 (d) $(-1978)^2 + 1977$

Q2. X and Y are two matrices such that the transpose of $(X - Y)$ is $\begin{bmatrix} 3 & -2 \\ 0 & 4 \end{bmatrix}$. If $Y = \begin{bmatrix} 1 & -5 \\ 3 & 1 \end{bmatrix}$, then which of the following is correct ?

(a) $X = \begin{bmatrix} -4 & 5 \\ 1 & 5 \end{bmatrix}$ (b) $X = \begin{bmatrix} 4 & -5 \\ 1 & 5 \end{bmatrix}$ (c) $X = \begin{bmatrix} -4 & -5 \\ 1 & 5 \end{bmatrix}$ (d) $X = \begin{bmatrix} 4 & -5 \\ -1 & 5 \end{bmatrix}$

Q3. Function *f* defined by $f(x) = 2^{-x}$ is strictly increasing when

(a) $x \in \phi$ (b) $x \in (-\infty, 0)$ (c) $x \in [0, \infty)$ (d) $x \in (-\infty, \infty)$

Q4. The points $A(1,2,3), B(-1,-2,-3)$ and $C(2,3,2)$ are three vertices of a paralleogram ABCD. The equation of CD is

(a) $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$ (b) $\frac{x+2}{1} = \frac{y+3}{2} = \frac{z-2}{3}$

(c) $\frac{x}{2} = \frac{y}{3} = \frac{z}{2}$ (d) $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-2}{3}$

Q5. The value of '*n*' such that the differential equation given by $\frac{dy}{dx} = \frac{x^3 + y^3}{x^n y}$; where $x, y \in R^+$ is homogeneous, is

(a) 0 (b) 1 (c) 2 (d) 3

Q6. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum}(i, j) - \text{minimum}(i, j)$:

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Q7. If A is a square matrix of order n, then the number of minors in the determinant of A are

(a) n (b) $n-1$ (c) n^2 (d) n^n

Q8. A bag contains 5 white and 3 black balls. 4 balls are successively drawn from the bag and not replaced, the probability that they are alternatively of different colours is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{1}{7}$ (d) $\frac{4}{7}$

Q9. There are two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a}.\vec{b}=0$. Then the projection of $\vec{a}$ on $\vec{b}$ is

(a) 1 (b) 0 (c) 2 (d) can not be determined

Q10. The graph of the inequality $2x+7y<40$ is the

(a) entire XY-plane
(b) open half plane that doesn't contain origin
(c) open half plane that contains origin, but not the points of the line $2x+7y=40$
(d) half plane that contains origin and the points of the line $2x+7y=40$

Q11. A and B are events such that $\mathrm{P}(\mathrm{A})=0.4$, $\mathrm{P}(\mathrm{B})=0.3$ and $\mathrm{P}(\mathrm{A}\cup\mathrm{B})=0.5$ then $\mathrm{P}(\mathrm{B}'\cap\mathrm{A})$ equals

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{10}$ (d) $\frac{1}{5}$

Q12. If $f(x)$ is continuous for all real values of x, then $\int_{\frac{a}{4}}^{\frac{b}{4}} f(4x)\,dx$ equals

(a) $4\int_{a}^{b} f(x)\,dx$ (b) $\frac{1}{4}\int_{4a}^{4b} f(x)\,dx$ (c) $\frac{1}{4}\int_{a}^{b} f(x)\,dx$ (d) $4\int_{4a}^{4b} f(x)\,dx$

Q13. The value of $\sec^2(\tan^{-1}3)+\cos ec^2(\cot^{-1}4)$

(a) 27 (b) 30 (c) 34 (d) 39

Q14. A differential equation has an order of 3 and a degree of 2. Which of the following could this differential equation be?

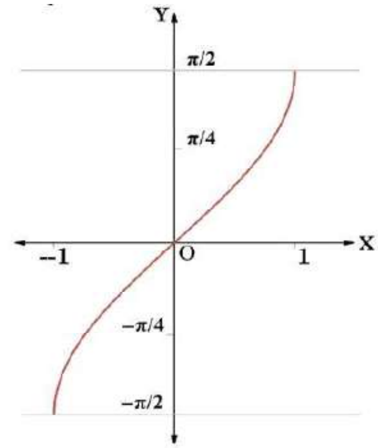
(a) $\frac{d^3y}{dx^3}-\left(\frac{d^2y}{dx^2}\right)^2=0$ (b) $\tan\left(\frac{d^2y}{dx^2}\right)+\left(\frac{d^3y}{dx^3}\right)^2=0$

(c) $\left(\frac{d^3y}{dx^3}\right)^2+\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3=0$ (d) $\left(\frac{d^3y}{dx^3}\right)^3+\left(\frac{dy}{dx}\right)^2=0$

Q15. The graph drawn below depicts

(a) $y = \cos^{-1} x$ (b) $y = \cos ec^{-1} x$

(c) $y = \sin^{-1} x$ (d) $y = \cot^{-1} x$



Q16. $f(x) = |\cos x|$ is non-differentiable at

(a) $x = n\pi,\ n \in Z$ (b) $x = (2n \pm 1)\pi,\ n \in Z$

(c) $x = (2n \pm 1)\frac{\pi}{2},\ n \in Z$ (d) $x = R - \left\{(2n \pm 1)\frac{\pi}{2}\right\},\ n \in Z$

Q17. If [.] denotes the greatest integer function, then $f(x) = [x]$ is discontinuous at

(a) infinite points, in $3 < x < 7$ (b) only three points, in $3 < x < 7$

(c) only five points, in $3 < x < 7$ (d) no point, in $3 < x < 7$

Q18. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between these vectors. Then $\vec{a} + \vec{b}$ is a unit vector if

(a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{3}$ (c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$

Assertion-Reason

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion :** The maximum value of the function $f(x) = x^5,\ x \in [-1, 1]$, is attained at its critical point, $x = 0$

Reason : The maximum value of a function can only occur at points where derivative is zero.

Q20. **Assertion :** If the angle between $\vec{p}$ and $\vec{q}$ is obtuse, then $\vec{p}.\vec{q} < 0$.

Reason : Value of $\cos\theta$ lies in $(-1, 0)$, when $90° < \theta < 180°$.

Section - B

Q21. If $4x + \frac{1}{x} = 4$, then find $\sin^{-1} x + \sin^{-1} 2x$.

Q22. The total cost of manufacturing 'x' smartphones per day in a Delhi based start-up XYZ Communication Limited is given by $C(x) = 0.0001x^2 + 4x + 2000$. Find $C(x = 50)$. Also, find the marginal cost of manufacturing 200 smartphones.

Q23. Differentiate $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ with respect to $\sqrt{1-a^2x^2}$, if $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$

OR

Differentiate w.r.t. x : $\cos^{-1}\left[\frac{\sqrt{1+\cos x} - \sqrt{1-\cos x}}{2}\right]$

Q24. UBER - CAB SERVICES are very popular in metropolitan cities. Ravi requested for a cab from Pitampura to Dwarka at 9:00 pm. He is to reach Dwarka in one hour (*i.e.* till 10:00 pm). Cab-driver may take 5 minutes to 15 minutes to pick the guest. After booking cab may be late over 15 minutes with probability $\frac{1}{20}$. The probability that guest will reach on time is 70%. What is the probability that Cab-driver reach on time to pick the guest from pitampura and drop him within one hour ?

Q25. Find out the area of the region enclosed by the curve $y = 2\sqrt{x}$, $x = 4$ and x-axis in the first quadrant.

<u>Section - C</u>

Q26. Find the point of intersection of the line $\vec{r} = \left(3\hat{i} + \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$ and the line passing through the point $(2, -1, 1)$ parallel to the z-axis. How far is this point from the z-axis?

Q27. Consider the following Linear Programming Problem.
Maximize $Z = x + 2y$
Subject to $2x + 3y \ge 6,\ 4x + y \ge 4;\ x,\ y \ge 0$. Show graphically that the maximum value of Z will not occur.

Q28. Evaluate : $\int \frac{x^{24}}{x^{10}+1}\, dx$

Q29. Solve : $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

Q30. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$

Q31. Differentiate w.r.t. x : $\sin^{-1}\left(x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right)$

OR

If $x^y \cdot y^x = 1$, prove that $\frac{dy}{dx} = -\frac{y(y+x\log y)}{x(y\log x + x)}$

Section - D

Q32. The curve $y = ax^2 + bx + c$; (where $a, b, c \in \mathrm{R}$ and $a \neq 0$) passes through the points $(-1,0),(2,12)$. Use matrix method to determine the values of a , b and c by solving the system of linear equations in a , b and c . Find the equation of the curve. If $y = ax^2 + bx + c = 0$, then write the real roots of quadratic equation (if possible).

Q33. Find the distance of the point $(3,5,7)$ from the line $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z+6}{-5}$ along the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

OR

Find the Cartesian equation of a line L_2 which is the mirror image of the line L_1 with respect to line $\mathrm{L}: \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line L_1 passes through the point P(1, 6, 3) and parallel to line L.

Q34. Determine the value of *a, b, c* for which the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{for } x < 0 \\ c, & \text{for } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{for } x > 0 \end{cases}$$

is continuous at $x = 0$.

OR

Find $\frac{dy}{dx}$: $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Q35. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point $(-4,-3)$. Find the equation of the curve given that it passes through $(-2,1)$.

OR

Show that the differential equation $2ye^{\frac{x}{y}}\,dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous and find its particular solution, given that , $x = 0$ when $y = 1$.

Section - E

Q36. Excessive use of digital devices can disrupt sleep patterns and lead to poor sleep quality. In a class of students aged 13 to 16 years, the students were grouped based on their daily screen time and its effect on their sleep.

* Group A: Spends more than 5 hours per day on screens (45% of students)
* Group B: Spends 3 to 5 hours per day on screens (35% of students)
* Group C: Spends less than 3 hours per day on screens (20% of students) A sleep quality survey found the following.
*75% of Group A students reported poor sleep quality
* 60% of Group B students reported poor sleep quality
* 20% of Group C students reported poor sleep quality. Using this information given above, answer the following questions.

(i) What is the total percentage of students who suffer from poor sleep quality?
(ii) A student is randomly selected and is found to have poor sleep quality. Find the probability that s/he belongs to Group A?

Q37. Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of cardboard of side 18 cm. Now, x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm. Based on the above information, answer the following questions :

(i) Express Volume of the open box formed by folding up the cutting corner in terms of x and find the value of x for which $\frac{dv}{dx} = 0$.
(ii) Sonam is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

Q38. A girl walks 3 km towards west to reach point A and then walks 5 km in a direction 30° east of north and stops at point B. Let the girl starts from O (origin) and take $\hat{i}$ along east and $\hat{j}$ along north. Based on the above information, answer the following questions.

(i) Find the scalar components of $\overrightarrow{AB}$
(ii) Find the unit vector along $\overrightarrow{AB}$
(iii) Find the position vector of point B.